Breakup morphology of inelastic drops at high Weber numbers

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Abstract

We report breakup morphology of inelastic non-Newtonian drops exposed to a co-flowing air stream at high air jet velocities. Power-law solutions were prepared by mixing one of two grades of CMC (sodium carboxymethyl-cellulose) with DI-water. Breakup morphology is reported in terms of Weber number, $We$ based on the air density, $\rho_a$, and Ohnesorge number, $Oh$, based on the zero shear rate viscosity. We present results for the sheet thinning and catastrophic breakup regimes, which correspond to $We$ beyond those at which bag-and-stamen breakup is seen. For the sheet thinning case the morphology is very similar to its Newtonian counterpart. For the catastrophic case, however, we observed the formation of a thin sheet which rolls up from it edges before collapsing onto itself and eventually disintegrating into smaller fragments. These topological changes are strongly dependent on the ratio $Oh/\sqrt{We}$. In contrast, the dependence on power law index, $n$, however, was not found to be strong for the range of $n$ tested.

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**Introduction**

Sprays are ubiquitous in everyday life and contain an ensemble of various sized drops. These distributions are pivotal in determining the efficacy of devices or conditions producing these sprays [1]. For instance, with pesticide sprays we wish to avoid spray drift and thus don’t want drops below a certain size. The goas for automobile engines is quite different, where a finer spray would lead to more efficient combustion. In addition, applications such as paint sprays require a more mono-dispersed drop size distribution.

Obviously, there is a need to understand the factors governing these drop size distributions in various scenarios. These factors may be broadly categorized as, (i) the geometry of the spray-producing device, (ii) transport properties of the liquid being atomized and, (iii) the environment in which the liquid atomization takes place. Amongst these, (i) and (ii) have been extensively studied so in this work we focused on analyzing the effect of viscosity for an inelastic non-Newtonian fluid. From a practical standpoint these shear rate dependent viscosity characteristics are introduced by performance enhancing additives like nitromethane or, n-butane in fuels such as gasoline. The use of gelled propellants displaying strong non-Newtonian characteristics are also employed in rocket engines for achieving greater thrust [2]. These reasons serve as the motivation for the study of non-Newtonian liquid fuel atomization.

Atomization of bulk fluid into smaller fragments occurs in stages [3]. The first of these is called primary atomization wherein the liquid fuel jet or sheet disintegrates into drops. Subsequent fragmentation of these drops is termed secondary atomization. Factors (i), (ii) and (iii) play an important role during primary and secondary atomization.

In the current study we investigate the effects of shear rate dependent viscous characteristics. These are introduced by performance enhancing additives like nitromethane or n-butane in fuels such as gasoline. The use of gelled propellants displaying strong non-Newtonian characteristics are also employed in rocket engines for achieving greater thrust [2]. These reasons serve as the motivation for the study of non-Newtonian liquid fuel atomization.

The measured rheologies are presented in Figures 1, 2, and 3. Drop strain rates were calculated to be between 1400 and 2300 1/s, which are within the range covered by the two rheometers. The Bird-Carreau rheological model was used because it can accurately model the Newtonian plateau observed at very low strain rates and the strong shear-thinning behavior at increased strain rates. Although a Newtonian plateau at high strain rates was not experimentally observed, \( \mu_\infty \)
was set to the viscosity of the base liquid (water) per [13]. Table 1 includes rheological parameters for all three solutions. Note that for DI-water, the Bird-Carreau parameters are taken to be $\mu_0 = \mu_\infty = 0.001$ Pa-s, $n = 1$, and $\lambda = 0$ s.

The surface tension of all three CMC solutions was determined using a du Noivy ring tensiometer. Values for the 0.8 wt.-% CMC-7MF ($\sigma = 0.0724 \pm 0.0005$ N/m) and 0.5 wt.-% CMC-7HF ($\sigma = 0.0727 \pm 0.0002$ N/m) solutions were close to the literature value of water ($\sigma = 0.0728$ N/m). One standard deviation from a sample size of ten was used as the measurement uncertainty.

Table 2 lists all experimental conditions in terms of $Oh$, $We$ and $\sqrt{We/Oh}$. Uncertainties were calculated in the standard manner using the Kline and McClintock method [2,4].

**Experimental methods and setup**

Figure 4 shows the experimental arrangement used to study the fragmentation processes. The main components are a needle, which serves as a drop generator, connected to a syringe pump which forces fluid to the needle through a polystyrene hose. The drops fall into the continuous jet that is produced by a specially designed air nozzle connected to a ~700 kPa (100 psi) air supply. Jet turbulence velocity fluctuations are within 3% of the mean and the velocity profile is nearly flat for most of its width. Drops produced by the needle are approximately 2.5 mm in diameter, with the exact diameter depending on the solution surface tension and viscosity.

Images were captured using a Vision Research Phantom v7 camera coupled to a 105 mm focal length lens that provide the necessary magnification. A 1000 W Xenon arc lamp (Kratos model LH151N) produced the required background illumination, which was followed by a plano-convex and optical diffuser so as to provide more uniform illumination. Images were recorded at 4700 fps with an exposure time of 100 $\mu$s and resolution of 800 $\times$ 600 pixels. This was adequate to capture the drop breakup dynamics.

**Results and Discussion**

The sheet thinning and catastrophic breakup patterns were observed. They are analyzed for inelastic shear thinning non-Newtonian drops.

Regime boundaries ranged from $30 \leq We \leq 50$ for sheet thinning breakup and $50 \leq We \leq 70$ for catastrophic breakup. As Figure 5 shows, they are relatively unaffected by $n$ for the range considered during this study, although a slight variation is seen for the range of $Oh$ considered.

Sheet thinning breakup is ascribed to Rayleigh-Taylor (R-T) instabilities. Figure 6 shows the different stages: (i) the initial spherical drop, (ii) the periphery of the drop separating out from the core because aerodynamic loading is becoming more important, and then (iii) the core flattening as it is gradually subjected to R-T destabilization. In the final stage we see complete fragmentation of the drop and long ligaments are formed. The long ligaments further destabilize owing to interactions with the surrounding air, and eventually lead to an ensemble of drops. This behavior is consistent with that observed for Newtonian drops [1], except for the effects of shear thinning.

The process for determining R-T instability wavelength is shown by the blue arrows in Figure 7. The maximum unstable wavelength for sheet thinning breakup, $\lambda_{st}$, non-dimensionalized by the initial drop diameter, $D_0$, is seen to vary exponentially with $Oh/\sqrt{We}$ in Figure 9. The dependence on $n$, which is a measure of the shear thinning behavior, is not apparent. It may lead to an increase in $\lambda_{st}/D_0$ at a given $Oh/\sqrt{We}$, further study is needed for verification.

Fragmentation is sudden in the catastrophic regime due to the higher aerodynamic loads imposed on the drops. As Figure 8 shows, the breakup processes proceed much the same as for sheet-thinning breakup through (iii). However, after this point the drop does not destabilize due to R-T instability, but is instead rapidly flattened into a sheet having a thin central region and thick rim. The rim retracts due to surface tension, rolls up, and then disintegrates into fragments. The collapse time, $\tau_{col}$ is chosen to quantify this process.

Figure 10 presents the dependence of collapse time non-dimensionalized by the characteristic time ($\tau = D_0\sqrt{\rho_l/\rho_0}$) for a liquid sheet in catastrophic breakup versus $Oh/\sqrt{We}$ for different values of $n$. The collapse time increase with increasing $Oh/\sqrt{We}$ is consistent with the assumption that viscosity lengthens breakup of liquid films and filaments. The dependence of this quantity on $n$ is not clear from the limited observation made, however one may surmise that the collapse time increases with increasing $n$ for a given $Oh/\sqrt{We}$.

**Summary and Conclusions**

This study determines the effects of high $We$ on the secondary atomization of the inelastic shear thinning drops. For the sheet thinning regime, $30 \leq We \leq 50$, the behavior was similar to that of Newtonian drops except for shear thinning viscous effects in the non-Newtonian case. In the catastrophic regime, $50 \leq We \leq 70$, a liquid sheet not seen in Newtonian drops was formed, rolled up into itself, then fragmented. The instability wave-
length and sheet collapse time for these regimes was quantified and their dependence on $Oh/\sqrt{We}$ established.

**Nomenclature**

- $\rho_a$ density of air $[kg\cdot m^{-3}]$
- $\rho_l$ density of the liquid $[kg\cdot m^{-3}]$
- $\sigma$ surface tension between air and liquid interface $[N/m]$
- $\mu_l$ dynamic viscosity of liquid drop $[Pa\cdot s]$
- $D_0$ initial diameter of the undeformed drop $[m]$
- $\lambda_{st}$ instability wavelength in the sheet thinning regime $[m]$
- $\tau$ characteristic breakup time in secondary atomization $[ms]$
- $\tau_{col}$ collapse time for the liquid sheet $[ms]$
- $U$ mean air jet velocity $[m/s]$

**References**

Table 1. Bird-Carreau rheological parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>0.5 wt. - % CMC - 7HF</th>
<th>0.8 wt. - % CMC - 7MF</th>
<th>1.4 wt. - % CMC - 7MF</th>
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<tr>
<td>$\mu_0$ [Pa-s]</td>
<td>0.576 ± 0.029</td>
<td>0.0596 ± 0.0030</td>
<td>0.309 ± 0.015</td>
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<tr>
<td>$\mu_\infty$ [Pa-s]</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>$n$ [-]</td>
<td>0.169 ± 0.008</td>
<td>0.427 ± 0.021</td>
<td>0.397 ± 0.020</td>
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<td>$\lambda$ [s]</td>
<td>0.334 ± 0.017</td>
<td>0.173 ± 0.009</td>
<td>0.324 ± 0.016</td>
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</table>

Table 2. Test conditions in terms of dimensionless parameters.

Uncertainty in $Oh = \pm 0.004$, $We = \pm 2$, $\sqrt{We/Oh} = \pm 2.5$

<table>
<thead>
<tr>
<th>Dimensionless number</th>
<th>Sheet thinning</th>
<th>Catastrophic</th>
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<tbody>
<tr>
<td>$Oh$</td>
<td>0.044 – 0.427</td>
<td>0.044 – 0.427</td>
</tr>
<tr>
<td>$We$</td>
<td>30 – 50</td>
<td>50 – 80</td>
</tr>
<tr>
<td>$\sqrt{We/Oh}$</td>
<td>12.82 – 160.70</td>
<td>16.55 – 203.27</td>
</tr>
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</table>

Figure 1. Viscosity versus strain rate of 0.5 wt.-% CMC-7HF.
Figure 2. Viscosity versus strain rate of 0.8 wt.% CMC-7MF.

Figure 3. Viscosity versus strain rate of 1.4 wt.% CMC-7MF.
Fig 4: Experimental setup for the study of the drop fragmentation process.

Figure 5: Regime boundaries for sheet thinning and catastrophic breakup based on the $We$ and $Oh$. Three $We$ are presented in the plot for a given $Oh$. Each $Oh$ corresponds to a particular $n$, the flow behavior index.
Figure 6. Breakup morphology for sheet thinning breakup and catastrophic breakup modes for the 0.5% CMC and D.I. water solution and $We = 33$ and $55$. Time difference, $\Delta t$ between successive images, (i), (ii), (iii), (iv) and (v) and (vi) for both types of breakup is roughly 2 ms. The changes in morphology of the drop are captured from its initial spherical shape to its final fragmented state.

Sheet thinning breakup

Catastrophic breakup
Figure 7. The formation and evolution of the liquid sheet in the catastrophic breakup regime for 0.8 % CMC and D.I. water solution at $We = 45$. Time difference, $\Delta t$ between successive images is roughly 2 ms. The red dotted circle shows the initial liquid sheet and the red arrows in subsequent images shows the retraction of the edges of the liquid sheet until their final collapse in the last frame.

Figure 8. Rayleigh-Taylor instability as observed in the breakup of 1.4 % CMC and D.I. water solution at $We = 40$ in the sheet thinning regime. Time difference, $\Delta t$ between successive is roughly 3 ms. The blue arrows indicate the crests and troughs corresponding to the Rayleigh-Taylor instability wavelength.

*Blue arrows: Rayleigh-Taylor instability wavelength*
Figure 9. Variation of non-dimensional Rayleigh-Taylor instability wavelength with \( Oh/\sqrt{We} \) for different values of power law index, \( n \). Uncertainty in measurements vary between 4 – 8%.

Figure 10: Variation of collapse time for liquid sheet in catastrophic breakup with \( Oh/\sqrt{We} \) for different values of power law index, \( n \). Uncertainty in measurements vary between 5.6 – 9.7%.