Computational study of heat transfer from a suspending needle through Rayleigh-Marangoni convection in an evaporating pendant droplet

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Abstract
The effect of suspender on the heat transfer in an evaporating microliter pendant droplet is studied numerically. Suspended pendant droplet from a needle serves as the easiest way to evaluate and compare evaporation rates of fuel droplets. However, this suffers from the intrusive effect of the suspender which is not present in the case of a free droplet. This study attempts to study this effect numerically. Steady state simulations are performed in an axisymmetric domain consisting of the needle and the droplet. Buoyancy-driven and Marangoni convection are included in the fluid domain and along the surface of the droplet respectively. The flow and energy equations are solved considering the exact shape of the droplet at a certain instance of evaporation using a commercial CFD package, ANSYS FLUENT 14.0. The surface temperature is prescribed from the measured data for the evaporation of pure liquid ethanol droplet (diameter ~ 3 mm) suspended using a hollow, flat-tip steel needle (length 120 mm and inner radius 2.6 mm) evaporating under atmospheric conditions inside a cubical chamber. The calculated heat transfer rates at the droplet surface from the simulation compare well with the experimental values.

(Keywords: Droplet evaporation, suspender, Marangoni convection)

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Introduction

Droplet evaporation has been an active area of research owing to the wide variety of applications it has ranging from spray drying to fuel combustion in gas turbine engines. Studies on isolated droplet evaporation under different conditions have helped determine evaporation characteristics of different liquids. One of the easiest ways of estimating and comparing evaporation rates of liquids is by using the pendant droplet method. Here a drop is suspended from a needle into a gaseous phase. The shape of the droplet is determined by the magnitudes of the force due to surface tension $\sigma$ and force due to gravity. Evaporation rates are measured by monitoring the droplet surface regression rate with time.

Although widely used, the pendant droplet method suffers from the intrusive effect of the suspender as opposed to free droplets. The effect of suspenders on evaporation rates of fuels have been studied qualitatively in earlier works [1,2]. Nomura et al. [3] studied the effect of initial droplet diameter ($d_0$) and suspender diameter on evaporation rates and natural convection. Using an evaporation rate corrected to eliminate the effect of the suspender, natural convection in evaporating droplets was studied. Yang and Wong, in separate studies [4,5], have reported agreement in values of evaporation rates between the experiment and the numerical model for an n-heptane droplet suspended in a weakly convective environment.

The presence of a suspender significantly increases the heat transfer to the evaporating droplet. Though the heat transfer is primarily by conduction through the suspender, studies have also included radiative heat transfer to the suspender and the liquid phase from the ambient [4]. Heat conduction is assumed to be one-dimensional in most cases with a spherical profile for the shape of the suspender/fiber bead and the droplet [3,6]. Also, none of the above studies considered the possibility of temperature variations on the droplet surface due to the heat transfer from the suspender through convection in the liquid phase. Shringi et al. [6] have reported significant temperature gradients at the point where heat enters the liquid through the fiber. This heat entering through the suspender causes temperature variations along the surface of the drop that leads to thermo-capillary convection or Marangoni convection in the liquid phase. This effect coupled with buoyancy driven convection in the liquid phase has been reported as a possible reason for internal circulation observed in experiments [7,8,9] on droplets evaporating under ambient conditions. The effect of flow in the ambient on the heat transfer into the droplet has been studied experimentally by Somasundaram [10]. Presence of flow outside an evaporating ethanol and a water droplet contributes to significant heat transfer by convection. This was found to be higher when a quartz fiber was used as a suspender instead of a metallic needle. This suggests that different suspenders have different rates of heat transfer into the droplet, depending on the ambient conditions.

Most of the studies mentioned, have assumed a constant temperature for the suspender and the droplet surface while calculating the heat transfer into the liquid. The effect of the ambient on the heat transfer to the evaporating droplet also has not been taken into account but for a few studies [4,6,10]. Assumptions on the shape of the droplet and the suspender have been introduced to reduce the complexity of the numerical simulations. This present study aims to simulate the heat transfer into an evaporating ethanol droplet by relaxing some of the assumptions on the droplet and suspender profile and by considering the effect of the ambient on the heat transfer. The results from experiments done in [10] shall be used for comparison with the present numerical work which is carried out using a commercial CFD package, ANSYS Fluent V14.0. The simulations are performed at steady state with the exact profile of the ethanol droplet used, at different stages of evaporation.

Computational domain

The experiments in [10] were performed in a cubical test chamber of side 0.24 m with provisions for inserting the metallic needle and the ethanol droplet. A steel needle of length 0.120 m, inner diameter of 0.0026 m and thickness 0.001 m was used. Simulations are performed on an axisymmetric domain consisting of the droplet and suspender. The solid and hollow portions of the needle are created as separate zones. A structured mesh with only quadrilateral elements was used to mesh the needle region. The needle was meshed with 6000 nodes in the axial direction along its length with the solid region having 10 nodes along the radial direction and the hollow region of the needle with 128 nodes in the radial direction. The mesh was efficiently graded to capture interactions near the fluid-solid interface and the region close to the axis of symmetry.

Figure 1. Computational domain of the solid and hollow regions of suspending needle
The droplet profile is created from images of the droplet at different stages of evaporation [10]. A structured mesh with quadrilateral elements was created with the Tri Primitive scheme using Gambit 2.4 and was used to mesh the droplet region. Figure 2 shows the mesh near the edge of the droplet surface and the suspending steel needle. Figure 3 shows the mesh generated for the ethanol droplet at different stages of evaporation (time $t = 10\text{s}, 70\text{s}$ and $140\text{s}$).

![Figure 2. Refinement of mesh (drop10s) near the base of the needle](image)

**Figure 2.** Refinement of mesh (drop10s) near the base of the needle

**Governing equations**

The governing equations in the steady state form are solved in the computational domain. Buoyancy forces are included in the momentum equation by applying the Boussinesq approximation for the liquid phase density. A linear approximation of liquid density given by $\rho = \rho_{ref} (1 - \beta(T - T_{ref}))$ is used where $\beta$ is taken as $1.4 \times 10^{-3} \text{K}^{-1}$ for ethanol.

\[
\begin{align*}
(\nabla \cdot \mathbf{V}) &= 0 \\
(\mathbf{V} \cdot \nabla) \mathbf{V} &= \frac{1}{\rho} \nabla p + \nabla \cdot (\mathbf{V} \nabla T) + \mathbf{g}
\end{align*}
\]

where $\rho$ is the density of the liquid phase, $k$ is the thermal conductivity in Wm$^{-1}$K$^{-1}$, $\nu$ is the kinematic viscosity in $m^2s^{-1}$ and $c_p$, the specific heat of the ethanol in the liquid phase in Jkg$^{-1}$K$^{-1}$. The inclusion of thermo-capillary convection due to variation of the surface tension with temperature is incorporated on the surface of the droplet as a boundary condition and will be described in the following section.

**Boundary Conditions**

The boundary conditions for the suspending needle and the droplet surface are applied considering the experimental conditions in [10]. In addition to those imposed by the axisymmetry the inner and bottom surfaces of the needle are treated as walls with no slip boundary condition for velocity and a *coupled* boundary condition for temperature since they interact with both solid and liquid regions. The top and outer surfaces of the needle are assigned a no-slip boundary condition for velocity with a constant temperature of $301 \text{K}$, the ambient temperature used in [10].

![Figure 3. Left: Droplet images taken at 10s, 70s and 140s after the start of evaporation; Right: Mesh generated after extracting droplet shape from the corresponding images](image)
in the experiments by Somasundaram [10] are provided as boundary condition for temperature at the droplet surface. Fig 3 indicates the variation of the droplet surface temperature \( T_S \) for the three cases considered at times 10s, 70s and 140s. It is seen from the figure that there is a steep gradient in the value of the droplet surface temperature close to the base of the needle. This concurs with a similar observation made by Shringi et al. [6] as mentioned earlier that further validates the choice of boundary condition.

**Solution Method**

The finite volume method is used for solving the conservation equations using the commercial CFD package ANSYS FLUENT V14.0. Flow and energy equations are solved in both the liquid and the solid regions separately. The suspending needle is made of steel with density, specific heat capacity and thermal conductivity as 8030 kg m\(^{-3}\), 502.48 J kg\(^{-1}\) K\(^{-1}\) and 16.27 W m\(^{-1}\) K\(^{-1}\) respectively. The thermo-physical properties of ethanol, the working fluid, are input. Density, specific heat capacity and thermal conductivity of ethanol are taken as 790 kg m\(^{-3}\), 2470 J kg\(^{-1}\) K\(^{-1}\) and 0.182 W m\(^{-1}\) K\(^{-1}\) respectively. The SIMPLE algorithm was used to resolve the coupling of pressure and velocity. A second order upwind scheme were used to discretize the momentum and energy equations. The under relaxation factors for the continuity, momentum and energy equation were retained as 0.3, 0.7 and 1 respectively. The temperature variation at the droplet surface was specified through user defined functions (UDFs) for each case.

**Results and Discussion**

Several studies earlier have shown the evidence of convection within the droplet when evaporating under different conditions. Therefore it is necessary to first establish if there is an increase in heat transfer because of the presence of convection. This is confirmed by solving the energy equation alone in the computational domain. The only mode of heat transfer in this case is by conduction through the solid and liquid regions. Secondly, buoyancy driven convection due to the temperature variation imposed on the surface is included in the liquid phase. It is seen that the convective velocities are very low (maximum velocity = 0.189 \( \times \) 10\(^{-3}\) m/s) and that there is hardly any increase in the heat transfer rate from the droplet surface (0.02368 W). Finally, the simulation is repeated with both the buoyancy and the Marangoni forces included. Comparison of the heat transfer rates across different surfaces in the domain for the test conditions is shown in Table 1.

All of the above simulations are performed on the domain consisting of the steel needle and the droplet at \( t = 10s \) after the start of evaporation. Interestingly, it is seen that the highest rate of heat transfer from the domain occurs for the case where only Marangoni convection is included. This suggests that the buoyancy driven convection and the Marangoni convection are counteractive resulting in a drop in heat transfer rate (0.12828 W) when combined Rayleigh-Marangoni convection is included. It is seen that the presence of convection significantly increases the heat transfer in an evaporating droplet by approximately 0.1 W for the \( t = 10s \) case and also for the entire duration of evaporation.

<table>
<thead>
<tr>
<th>Test Condition</th>
<th>Heat Transfer Rates at various surfaces in the computational domain ( \times 10^{-3} ) W</th>
<th>Maximum velocity ( \times 10^2 ) m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Only Conduction</td>
<td>-23.61  23.63  8.16  15.47</td>
<td>-</td>
</tr>
<tr>
<td>Only Buoyancy</td>
<td>-23.68  23.68  8.20  15.48</td>
<td>0.0189</td>
</tr>
<tr>
<td>Buoyancy &amp; Marangoni</td>
<td>-128.28 128.28 110.60 17.68</td>
<td>8.92</td>
</tr>
<tr>
<td>Only Marangoni</td>
<td>-136.65 136.65 119.20 17.45</td>
<td>8.97</td>
</tr>
</tbody>
</table>

**Table 1.** Comparison of heat transfer rates at various surfaces using different modes of convection (time \( t = 10s \))
A droplet evaporating under ambient conditions has multiple sources of heat for evaporation. They are listed in [10] as follows: the latent heat of vaporization of the drop \( Q_{\text{latent}} \), the heat transfer from the suspender \( Q_{\text{con}} \), heat transfer from the ambient through convection \( Q_{\text{conv}} \) and through radiation \( Q_{\text{rad}} \). A simple energy balance shows that the heat transfer through the suspender is \( Q_{\text{evap}} = Q_{\text{conv}} + Q_{\text{rad}} \). This heat enters into the liquid phase and is also the net heat transfer at the droplet surface. Therefore a comparison is made between the heat transfer rates reported at the droplet surface between the experiment and the numerical simulation. For the present case of ethanol droplet suspended from a steel needle, experiments report the different heat transfer rates as follows: 85.09 x 10\(^{-3}\) W as \( Q_{\text{evap}} \), with 5.22 x 10\(^{-3}\) W contribution from convection \( Q_{\text{conv}} \) and 0.85 x 10\(^{-3}\) W from radiation \( Q_{\text{rad}} \). This implies the heat transfer into the liquid from the suspender \( Q_{\text{evap}} \), is 79.82 x 10\(^{-3}\) W. In addition to the heat transfer rate, the numerical simulation also gives us the distribution of the temperature and the velocity fields inside the ethanol droplet. Table 2 summarizes the heat transfer rates at the different surfaces in the computational domain in comparison with the experimentally measured heat transfer rate and maximum velocities in the domain.

It is seen that the results of the numerical simulation overestimate the heat transfer rate from the droplet surface. The reason for such over prediction could be a steeper gradient in temperature near the base of the needle than what is actually present. The difference in temperature between the outer surface of the needle (301 K) and the droplet surface temperature \( T_3 \) could be lesser than that of the values imposed in boundary condition. In addition to this, the mismatch could be due to the errors associated with the measurement of temperatures in the experimental study [10]. It is also seen that there is a significant increase in transfer rate between time \( t = 10\) s and \( t = 70\) s and 140 s respectively. This could be attributed to the temperature variation imposed on the droplet surface for the latter cases. It is seen that the surface temperatures of the droplet are much lower for the \( t = 70\) s and 140 s cases than at the beginning of evaporation when \( t = 10\) s. The decrease could be explained by the increasing effect Marangoni convection at the droplet surface which tends to even out the temperatures in the later stages of evaporation. A look at the maximum velocities reported in Table 2 justifies this argument as the velocities drop from 0.1078 m/s to 0.0907 m/s at \( t = 70\) s and 140 s respectively.

Table 2 shows that the heat transfer through the base of the needle is between 8 to 14% of the total heat transfer into the domain. This indicates that heat transfer within the suspending needle is two-dimensional and the assumption of a one dimensional conduction model would not estimate, correctly, the heat transfer into the liquid phase.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Heat Transfer Rates at various surfaces in the domain ((x \times 10^{-3} \text{ W}))</th>
<th>Maximum velocity ((\times 10^{-2} \text{ m/s}))</th>
<th>Heat transfer rate from experiment ([10]) ((x \times 10^{-3} \text{ W}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>-128.28 \hspace{1cm} 128.28 \hspace{1cm} 110.60 \hspace{1cm} 17.68 \hspace{1cm} 8.92</td>
<td></td>
<td>-79.82</td>
</tr>
<tr>
<td>70</td>
<td>200.62 \hspace{1cm} 200.61 \hspace{1cm} 183.89 \hspace{1cm} 16.73 \hspace{1cm} 10.78</td>
<td></td>
<td></td>
</tr>
<tr>
<td>140</td>
<td>200.11 \hspace{1cm} 200.10 \hspace{1cm} 179.35 \hspace{1cm} 20.75 \hspace{1cm} 9.07</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Comparison of heat transfer rates between experiment [10] and numerical simulation.
Two distinct recirculation zones are seen due to this flow field and they migrate toward the centre of the drop during the later stages of evaporation. The temperature variation on the droplet surface slowly penetrate into the bulk liquid as evaporation occurs due to the above fluid motion. This is evident from the temperature contours at the different stages of evaporation.

Figure 6. Contours of temperature and velocity within the droplet at different stages of evaporation; Top to bottom: $t = 10s$ and $t = 70s$ and $140s$
Conclusions

A numerical study on the heat transfer from a suspending needle into an evaporating pendant droplet has been performed. For the case of an ethanol droplet, suspended from a steel needle, evaporating under ambient condition, it has been found that there is significant heat transfer into the drop when both buoyancy and Marangoni convection are considered in the liquid phase.

Heat transfer rates were found to be low if only conduction and/or buoyancy driven convection were used. The heat transfer rate at the droplet surface at different stages of evaporation viz., \( t = 10s \), \( t = 70s \) and \( t = 140s \) have been found out. It is seen that these values compare well the experimentally measured values at the \( t = 10s \) but are over estimated in the \( t = 70s \) and \( t = 140s \) case. The reasons for the mismatch may possibly be due to the extremely high temperature gradients at the base of the needle due to the imposed boundary conditions. It is also concluded that the conduction within the steel suspender is two dimensional with significant heating along both its inner and the bottom walls.

The temperature and velocity distributions within the droplet have been calculated. It has been concluded that the Marangoni shear stress induced surface flows strongly influences the velocity and temperature profiles in the bulk liquid. This effect is found to increase initially and then decreases in the later stage of evaporation as the temperature field within the droplet become uniform.

References