Parametric Study of HRM for Gasoline Sprays

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Abstract
Flash boiling is known to be a common phenomenon for Gasoline Direct Injection (GDI) engine sprays. Homogeneous Relaxation Model has been adopted in many recent numerical studies for predicting cavitation and flash boiling. Assessment of the Homogeneous Relaxation Model has been presented in this study. Sensitivity analysis of the model parameters has been documented to infer the driving factors for the flash boiling predictions. The model parameters have been varied over a range and the differences in predictions for the extent of flashing have been studied. Apart from flashing in the near-nozzle regions, mild cavitation is also predicted inside the gasoline injectors. The variation in the predicted time-scales through the model parameters for predicting these two different thermodynamic phenomena (cavitation, flash) have been elaborated in this study.

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Introduction

Comprehensive understanding of fuel spray formation is warranted for optimization of existing technologies, as well as future developments, in the field of diesel or gasoline direct injection combustion engines. Gasoline Direct Injection (GDI) engines have been preferred to Port Fuel Injection (PFI) engines for almost a decade. Experimental studies have shown that GDI offers better fuel economy and superior performance, when compared to PFI [1, 2]. The GDI engines become advantageous owing to elegant technologies such as, intelligent piston design, swirl flows, high pressure fuel injection, various injection strategies, exhaust gas recirculation (EGR), closed-coupled catalyst etc.

Flash boiling, can happen in modern GDI engines at throttled conditions and significantly affects spray formation [3]. Throttled operating conditions create low in-cylinder chamber pressures (≈ 30 kPa). Therefore, when heated gasoline-type fuel (around 363 K) is injected to such in-cylinder environments, liquid fuel is subjected to a superheated condition. Based on thermodynamic definition, superheated conditions arise when any liquid substance experiences ambient pressure lower than the saturation pressure, corresponding to its temperature. Flash boiling or flashing causes rapid bulk conversion of liquid fuel to gaseous vapor leading to volume expansion and thus, affecting spray width and penetration. Consequently, flashing can either be advantageous or disadvantageous depending on chamber design and injection timing as well as injector orientation. As a result flash boiling (or flashing) for GDI engines is becoming a relevant topic of research.

Apart from flashing there are two more phenomena involving phase change of liquid fuel to gaseous vapor - vaporization and cavitation. Flashing is typically a near-nozzle exit phenomenon. High temperature environment in combustion chamber can provide convective heating of liquid fuel causing “vaporization”. The liquid fuel enters the injector holes through the space between moving needle and needle sac. This resembles the flow from a large cross-sectional area to a much smaller cross-sectional area, where flow separation occurs near the entry of the smaller cross-sectional portion. Inside fuel injector holes, the local pressure in this flow separation region may drop below the saturation pressure of the liquid fuel, depending on local temperature, injection pressure and injector geometry. Such phenomenon is termed as “cavitation”. Cavitation occurs almost instantaneously, since it is typically a near-equilibrium process involving negligible heat transfer [4]. Cavitation is usually prominent in diesel injector holes, because injection pressure in diesel injector (upto 300 MPa) can be at least one order of magnitude higher than that of a gasoline injector. Flashing is usually perceived as non-equilibrium phenomenon, since heat transfer is non-negligible and requires a finite time-scale (relative to cavitation) for the phase change to occur [5].

Some experimental studies have been undertaken to investigate flash boiling effects on GDI spray patterns. VanDerWege and Hochgreb [6] observed in the Planar laser-induced fluorescence (PLIF) and Mie-scattering images the transition of a pressure-swirl injector spray from hollow-cone shape to solid-cone shape with increasing the superheat of the liquid fuel. The authors concluded from the image intensity comparisons that the droplet diameter reduction due to flash boiling is in the order of 40%. Based on the bubble-point calculation, they also suggested that a superheating by roughly 20 K is required for the flash boiling to generate noticeable effects. Their experimental results demonstrated the potential advantages of using flash boiling to enhance spray atomization. Zeng et al. [7, 8] studied experimentally the spray from a multi-hole injector using Mie-scattering and laser induced exiplex flourescense (LIEF). The authors quantified the macroscopic spray structure with penetration length, plume width and normalized plume distance. They also characterized the different macroscopic spray behaviors based on the ratio of ambient pressure to saturation pressure. A set of empirical expressions were proposed to predict the macroscopic structure of the spray under the effects of flash boiling. In the analysis, the authors focused more on the macroscopic spray structure rather than fundamental bubble and droplet dynamics. Weber and Leick [3] carried out high-speed Mie-scattering for global spray characterization and investigation of near nozzle spray structures using high-speed shadowgraphy. Additionally, droplet velocities were estimated by shadow particle image velocimetry technique. They observed that individual spray plume length reduces and the initial cone angle increases due to flash boiling. They noticed that during intense flash boiling situations two adjacent spray plumes might interact with each other. In a recent study with a GDI injector, similar to Spray G [9], a comprehensive experimental analysis was documented [10]. They used a Delphi Valve Covered Orifice (VCO) injector with iso-octane fuel. The injection pressure, the fuel temperature and the chamber pressure were varied, with the chamber temperature remaining at 296 K. They observed that sub-atmospheric chamber pressure and elevated fuel temperatures led to large at-
The time-scale evaluates the temporal extent of the phenomenon, which estimates the time-scale of phase change. The relaxation model (HRM) [17, 18, 19, 20] represents the phase transition by one empirical equation. Based on empirical coefficients, HRM is the homogeneous equilibrium model (HEM) and homogeneous frozen model (HFM) [18, 4]. In case of HEM, the two-phases are assumed to be mixed perfectly homogeneous with the heat transfer occurring spontaneously. In a real-world scenario of two-phase flows, such as bubbly flows, instantaneous heat transfer is not feasible. The other extreme, HFM assumes zero heat transfer or heat transfer time-scale to be infinitely long. HRM captures the in-between practical two-phase flow scenarios. The vapor-liquid equilibrium properties are necessary for HRM implementation and have been estimated for pure as well as blended fuels, for GDI applications [21, 5]. Recently, Moulai et al. [22] and Saha et al. [23] carried out numerical studies of iso-octane flash boiling in a multi-hole GDI fuel injector (Spray G) with conditions specified in the Engine Combustion Network (ECN) [9]. These studies examined cases where outlet chamber is filled with only iso-octane as well as cases where chamber is occupied solely by nitrogen gas. They explored different flashing and non-flashing conditions in their work.

HRM has been successfully used for modeling cavitation in diesel injectors in the recent past by many researchers [24, 25, 26, 27, 28, 29, 30]. HRM is a useful model for the industry, because of its utility in both diesel and gasoline direct injection sprays. It is vital to point out that the model rate equation and values of model constants in HRM have been same for solving flashing as well as cavitating problems. This raises a concern about the applicability of HRM for modeling two thermodynamically different phenomena such as, cavitation and flashing. Therefore, the objective of the current study is to provide in-depth understanding of HRM, pros and cons of the model setup and explore potential justifications of using HRM in flashing, non-flashing or cavitating conditions. The operating conditions and fuel injector considered in this study are based on the baseline Spray G condition [9].

Significant effort has been already made in terms of numerical studies on flash boiling phenomenon. They can be broadly classified into two-types: a) bubble dynamics based models, and b) empirical coefficients based thermodynamic rate equation models. Blinkov et al. [11] assumed mono-disperse bubble population and their model consisted of 5 transport equations including vapor formation, interface heat transfer, and nucleation. Blinkov et al. employed their model to analyze flashing and choking water flows and inferred that bulk nucleation becomes increasingly significant with increment in the volume to surface ratio of the geometry. Chang and Lee (2002) [12] modified Blinkov’s approach by considering poly-disperse bubble population in a converging water nozzle and concluded from their primary analysis that capturing the bubble size distribution is important for multiple and distributed sources of bubble nucleation. Kawano et al. (2004) [13] incorporated bubble growth, bubble perturbation and breakup to model the flashing and non-flashing sprays. A similar approach was adopted by Zeng and Lee (2001) [14] to study n-pentane sprays, subjected to different degrees of superheat. They concluded that more flashing can be correlated with increment in superheat, smaller drop sizes (∼50% reduction in SMD, when flashing is considered in the model), wider sprays and even plume merging. In this work, Zeng and Lee studied the effect of two competing mechanisms of secondary breakup - bubble growth and aerodynamic force, in a hollow cone spray issuing out of a pintle injector. They inferred that for lower degrees of superheat aerodynamic forces dominate, while bubble growth takes over at higher degrees of superheat. Zuo et al. [22] and Saha et al. [23] carried out numerical studies of iso-octane flash boiling in a multi-hole GDI fuel injector (Spray G) with conditions specified in the Engine Combustion Network (ECN) [9]. These studies examined cases where outlet chamber is filled with only iso-octane as well as cases where chamber is occupied solely by nitrogen gas. They explored different flashing and non-flashing conditions in their work.

Another type of flash boiling model based on empirical coefficients is the Homogeneous Relaxation Model (HRM) [17, 18, 19, 20] which represents the phase transition by one empirical equation, which estimates the time-scale of phase change. The time-scale evaluates the temporal extent of the deviation of the local condition from thermal equilibrium.

HRM lies in between the two extremes of thermodynamic two-phase models - homogeneous equilibrium model (HEM) and homogeneous frozen model (HFM) [18, 4]. In case of HEM, the two-phases are assumed to be mixed perfectly homogeneous with the heat transfer occurring spontaneously. In a real-world scenario of two-phase flows, such as bubbly flows, instantaneous heat transfer is not feasible. The other extreme, HFM assumes zero heat transfer or heat transfer time-scale to be infinitely long. HRM captures the in-between practical two-phase flow scenarios. The vapor-liquid equilibrium properties are necessary for HRM implementation and have been estimated for pure as well as blended fuels, for GDI applications [21, 5].
parameters will be discussed.

Model Formulation

The problem considered in this study is iso-octane fuel flowing through a 8-hole counter-bored GDI fuel injector, which is denoted as Spray G nozzle in the ECN [9]. Details of the cases studied in this work are summarized in Table 1. From prior work on Spray G cases by our group [23] and other research groups [22] it has been evident that Spray G condition is supposed to be non-flashing, since the chamber pressure is higher than the saturation pressure corresponding to iso-octane (liquid) fuel temperature. However, the chamber temperature being higher than the fuel temperature, vaporization of the liquid jets is expected due to convective heating from the hot environment. In case of Spray G2 flashing is expected because the chamber pressure is lower than the corresponding saturation pressure of the fuel. For Spray G2 the chamber is considered at room temperature, therefore, vaporization is not expected. The Spray G3 condition has been introduced for the first time in this study. Spray G3 has a higher fuel temperature and is subjected to chamber pressure of 1 atmosphere. Thus, the degree of superheat is higher than that of Spray G2 indicating possibility of more than mild/moderate flashing. The chamber temperature of Spray G3 is at room temperature, hence, there is no possibility of vaporization of liquid jets for Spray G3.

0.1 Nozzle Geometry and Computational Domain

The internal geometry of Spray G nozzle is not symmetric since it has 8 holes and 5 dimples which results in uneven upstream flow passages leading to counter-bored, i.e., stepped holes. The nominal diameters of the holes and the counter-bores are 165 \( \mu \text{m} \) and 388 \( \mu \text{m} \), respectively. The length to diameter ratio of the nozzle holes and counter-bores are around 1 and 1.2, respectively. Therefore for realistic predictions, full nozzle geometry has been simulated in this study. The nominal geometry of the Spray G nozzle has been obtained through the ECN. Figure 1 shows the location of the 8 stepped holes with respect to the 5 dimples. The upstream asymmetry in the nozzle geometry is evident from this figure. The information about the orientation of the holes is vital for cases where hole-to-hole variations become conspicuous inside the nozzle and/or in the near-nozzle regions. A hemispherical outlet domain is needed to ensure that the near-nozzle flow features are modeled accurately. It is difficult to have a correct \textit{a priori} estimation of the extent of the outlet domain. Results using outlet domain of 9 mm diameter have been reported in this study. Previous study [23] indicated that results using 9 mm outlet domain could be reasonable for internal and near-nozzle flow analysis at a physical time-stamp of 0.1 ms.

Simulations are performed using the CONVERGE code [31], which uses a cut-cell technique to generate the mesh automatically during the run time. A vertical cut-plane showing the mesh with 9 mm outlet domain and 17.5 \( \mu \text{m} \) as minimum grid size is shown in Fig. 2. Adaptive Mesh Refinement (AMR) has not been utilized to ensure efficient load-balancing while executing parallel computing with millions of cells in the computational domain. Fixed embedding has been used near the walls, inside the holes and in the near-nozzle regions, and inside the chamber to capture the sharp gradients in species, velocity, temperature, etc. Three levels of embedding have been used which means smallest cell size has a dimension \( 2^3 \), i.e., 8 times smaller than the...
largest cell size (base grid size) in the domain. In this work, results using 140 μm as base grid i.e. 17.5 μm minimum grid size has been presented, because this grid resolution have been shown to provide reasonable estimates with affordable wall-clock times [32].

0.2 Governing Equations

The mixture multiphase model in the Eulerian framework has been used for this study. The numerical setup for flash boiling requires solving the set of governing equations of mass, momentum, species and energy conservation, additional scalar equations for turbulence as well as constitutive equations for liquid and vapor phases to account for variable density due to pressure changes. The governing equations adopted in this study are available in the literature [[31, 25, 33, 26, 27]] and are provided here as well for the convenience of the readers. The CONVERGE code is used for the simulations and the mass, momentum and energy conservation equations are as follows:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial u_j \rho}{\partial x_j} = S
\]  
\[
\frac{\partial u_i}{\partial t} + \frac{\partial (u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial \sigma_{ij}}{\partial x_j} + S_i
\]  
\[
\frac{\partial \rho e}{\partial t} + \frac{\partial (u_i \rho e)}{\partial x_i} = -\frac{\partial}{\partial x_j} \left( \sigma_{ij} + \frac{\partial u_j}{\partial x_j} \right) + \frac{\partial}{\partial x_j} \left( \rho \Sigma_m D h_m \frac{\partial Y_m}{\partial x_j} \right) + \frac{\partial}{\partial x_j} \left( k \frac{\partial T}{\partial x_j} \right) + S_e
\]  

where,

\[
\sigma_{ij} = \frac{\partial}{\partial x_j} \left[ \left( \mu + \mu_t \right) \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial u_i}{\partial x_i} \right) \right]
\]

and \( \mu \) is the molecular viscosity and \( \mu_t \) is the turbulent viscosity which is modeled as:

\[
\mu_t = C_{\mu k} \frac{k^2}{\epsilon}
\]

using the standard \( k-\epsilon \) model. The mixture density is calculated from:

\[
\rho = \sum \rho_m \alpha_m
\]

and

\[
\alpha_m = Y_m \frac{\rho}{\rho_m}
\]

where \( \rho \) is the mixture density and \( \rho_m \) stands for the density of any constituent species of the mixture. The species considered are - liquid fuel, vapor fuel, \( N_2 \) and \( O_2 \). The densities of all the gaseous species have been estimated using the Ideal Gas Equation. \( \alpha_m \) and \( Y_m \) represent the volume fraction and mass fraction of the individual species. The individual mass fractions are determined from the species conservation equations:

\[
\frac{\partial \rho Y_m}{\partial t} + \frac{\partial (u_j \rho Y_m)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \rho D \frac{\partial Y_m}{\partial x_j} \right) + S_m
\]

where \( S_m \) represents the source or sink term. There are no source or sink terms for non-condensable gases (\( N_2 \) and \( O_2 \)). \( S_v \) and \( S_l \) are the source terms for vapor fuel and liquid fuel transport equations. \( S_v \) provides the rate of phase change for cavitating, flashing and vaporizing conditions. Consequently, we have \( S_v = -S_l \). The relevant details for estimating \( S_v \) are provided in the “Homogeneous Relaxation Model” sub-section.

0.3 Turbulence Model

The turbulent kinetic energy \( k \) and the turbulent dissipation rate \( \epsilon \) are determined from the turbulence model. The standard \( k-\epsilon \) turbulence model has been used. The conservation equations are:

\[
\frac{\partial k}{\partial t} + \frac{\partial (u_j k)}{\partial x_j} = \tau_{ij} \frac{\partial u_i}{\partial x_j} + \frac{\partial}{\partial x_j} \left( \mu \frac{\partial k}{\partial x_j} - \rho \epsilon \right)
\]

Table 1. Cases Studied

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Spray G</th>
<th>Spray G2</th>
<th>Spray G3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inj. Press. (MPa)</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Chamber Press. (kPa)</td>
<td>600</td>
<td>53</td>
<td>100</td>
</tr>
<tr>
<td>Chamber Temp. (K)</td>
<td>573</td>
<td>293</td>
<td>293</td>
</tr>
<tr>
<td>Fuel Temp. (K)</td>
<td>363</td>
<td>363</td>
<td>413</td>
</tr>
<tr>
<td>Chamber Fluid</td>
<td>N₂</td>
<td>N₂</td>
<td>N₂</td>
</tr>
<tr>
<td>Degree of Superheat (ΔT) (K)</td>
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<td>12.34</td>
<td>40.68</td>
</tr>
<tr>
<td>Pressure Ratio (R_P)</td>
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<td>1.48</td>
<td>2.83</td>
</tr>
<tr>
<td>Thermodynamic State</td>
<td>Non-flashing</td>
<td>Moderate</td>
<td>Intense</td>
</tr>
<tr>
<td></td>
<td>Vaporizing</td>
<td>Flashing</td>
<td>Flashing</td>
</tr>
</tbody>
</table>
\[
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_j}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \mu \frac{\partial \rho}{\partial x_j} \right) + C_{e3} \rho \frac{\partial u_j}{\partial x_j} + \frac{\partial}{\partial x_j} \left( C_{e1} \frac{\partial u_j}{\partial x_j} \tau_{ij} - C_{e2} \rho \epsilon \right) \frac{\epsilon}{\kappa} \tag{10}
\]

where Reynolds’s stress is:

\[
\tau_{ij} = -\bar{p} u_i^j u_j = 2 \mu_t S_{ij} - \frac{2}{3} \delta_{ij} \left( \frac{\rho k + \mu_t}{\rho} \frac{\partial u_i}{\partial x_j} \right)
\]

The model constants are as follows:

\[
C_{\mu} = 0.09; \quad C_{e1} = 1.44; \quad C_{e2} = 1.9; \quad C_{e3} = -1.0; \quad 1/Pr_t = 1.0; \quad 1/Pr = 0.77
\]

### 0.4 Homogeneous Relaxation Model

The source term \(S_v\) in the vapor species conservation equation is evaluated through the HRM formulation. The rate of change of local vapor quality is evaluated through the HRM scheme

\[
S_v = \frac{(x^1 - x^0) (m_{\text{vap}} + m_{\text{liq}})}{V \Delta t} = (x^1 - x^0) \frac{(Y_v + Y_l)}{V \Delta t} \tag{13}
\]

where, \(V\) is the volume of the computational cell and \(m_{\text{vap}}\) and \(m_{\text{liq}}\) are mass of vapor and liquid phases respectively in that computational cell. The vapor quality is mathematically represented as:

\[
x = \frac{Y_v}{Y_v + Y_l} = \frac{m_{\text{vap}}}{m_{\text{vap}} + m_{\text{liq}}}
\]

### 0.5 Boundary and Initial Conditions

Pressure is identified at the inlet and outlet of the nozzle, and no-slip boundary condition is considered at the walls. No-slip condition is also assumed at the interface of liquid and vapor for the mixture multiphase approach. Turbulence boundary conditions are specified by the turbulent kinetic energy and turbulence dissipation rate at the inlet and the outlet.

A liquid fuel cannot be 100% pure, therefore, it is reasonable to consider the presence of very small amount \((\approx 10^{-5}\) mass fraction\) of non-condensable gases [28, 34], which acts like nucleation-sites for cavitation initiation. The volume fraction of the gaseous phase obtained from the solution includes both the vapor and non-condensable gases. Initially no vapor is present in the computational domain. However, non-condensable gases are present in the liquid fuel in very small mass fractions \((N_2 = 10^{-5}; O_2 = 10^{-5})\). The chamber is initialized with chamber pressure, chamber temperature and only nitrogen. The 8 holes along with counter-bores, are initialized with chamber pressure, fuel temperature, and 99.998% liquid fuel and trace amounts of non-condensable gases as mentioned before. The needle seat and the upstream of sac region up to the inlet are initialized with a pressure of fuel injection pressure, fuel temperature and 99.998% liquid fuel and trace amounts of non-condensable gases. Simulations are transient, and all calculations were run until both the inflow and the outflow mass flow rates are stabilized.

### 0.6 Solution Procedure

The commercial software, CONVERGE v2.2, is used in the current study to solve the governing equations using the finite volume approach. Rhie-Chow algorithm has been adopted to have all the transport variables collocated at the cell center. Pressure implicit with splitting of operator (PISO) algorithm is used for pressure-velocity coupling, since it is expected to work well for unsteady compressible flow problems [31]. For momentum equation second order central differencing is employed, while for other transport equations first order upwind scheme has been applied. Successive Over-Relaxation (SOR) algorithm has been observed to offer better numerical stability. The flash boiling problem studied in this work is a two-phase compressible flow where sharp or nearly sharp interface between liquid and vapor phases is not expected. As a result, both piecewise-linear interface calculation (PLIC) and high-resolution interface capturing (HRIC) schemes have been de-activated for the
present set of simulations. PLIC is known to be effective for only incompressible flows and HRIC is expected to be useful for very sharp density changes such as compressible cavitating flows in high pressure diesel injector [25]. Flash boiling conditions will have more smeared two-phase distribution with a diffused interface [23, 32].

0.7 MATERIAL PROPERTIES

Majority of the material properties, utilized in this study, are obtained from the material database of the code [31]. Modifications were required for capturing liquid density changes with changes in pressure and temperature. For implementing liquid compressibility, reference pressure, reference density and bulk modulus information are incorporated using the data available in the literature [35].

1 Results and Discussions

In this section, first the effect of thermodynamically different boundary conditions will be demonstrated through HRM predictions. Then, results from parametric study will be presented.

1.1 Boundary & Operating Conditions

In the current study, we do not show any validation of the simulation setup. Our previous studies [23, 32] show rigorous validation and hence is not shown here for the sake of brevity. The different conditions studied are Spray G, Spray G2 and Spray G3 and are shown in Table 1. The expected outcomes for these 3 conditions have been already stated prior to this section. Figure 3 presents the vapor mass fraction contours at 0.1 ms. The predicted results are in accordance with prior estimation based on thermodynamic analysis. Spray G calculation indicates that the periphery of the liquid jets are vaporizing due to convective heating in the high temperature chamber. Spray G2 is moderately flashing, since the degree of superheat is not very high. Degree of superheat is very high for Spray G3 and hence there is abundant vapor formation, i.e., intense flashing in the chamber and significant plume-to-plume interaction, as shown in the simulation result. It is vital to point out that HRM is used to model any sort of phase change in the simulations. Moreover, HRM has not been proven in the literature to be effective for vaporization caused by high ambient temperature. Therefore, vapor formation prediction in Spray G may not necessarily be an accurate representation of the actual non-flashing but vaporizing scenario.

The above result and corresponding discussion only depicts how the mass fraction contours vary, when subjected to different thermodynamic conditions. It is vital to keep in mind that density of vapor fuel is order of magnitude lower than that of liquid fuel. Therefore, the volume fraction of vapor fuel is going to be significantly higher than the mass fraction values. Figure 4 presents the mass fraction and volume fraction contours of vapor iso-octane for Spray G2 case. As a result, it is clear that, even though not considerable mass of vapor is formed at moderate flashing, the vapor phase occupies substantial volume in the counter-bores and farther downstream along the spray plumes.

1.2 HRM Parametric Study

The constants of the HRM have been optimized for cases where water flowing through a nozzle is flashing [17, 18]. In the current study, the fluid, operating and boundary conditions are different from [18] wherein these model constants were optimized through curve-fitting with measured data for water. Hence, analyzing the sensitivity of the model predictions with respect to the model constants is warranted in studies involving different fluids with HRM. The time-scale constant \( \theta_0 \) and exponents of \( \alpha \) and \( \psi \) from Eq. 12 are varied for these sensitivity studies. \( \theta_0 \) has been varied from the order of \( 10^{-6} \) to \( 10^{-8} \). Figure 5 presents the effect of changing \( \theta_0 \) on vapor formation for Spray G2 condition. Lower value of \( \theta_0 \) seems to trigger more flashing which is expected since \( \theta_0 \) considerably lowers the equilibrium time-scale, leading to significant increment of \( \frac{Dx}{Dt} \) throughout the run-time. Therefore, it is evident that \( \theta_0 \) is a very influential factor, at least for the case of moderate flashing. Intense flashing is expected when pressure ratio \( (R_P) \) is greater than 2 or 3 [3] and for Spray G2 \( R_P < 2 \). Therefore, Spray G2 should be moderately flashing and \( \theta_0 \approx 10^{-7} \) would still seem to be reasonable for Spray G2 condition.

Exponent of \( \alpha \) has been varied from -0.27 to -0.81 with the default value being -0.54. The effect of variation is shown in Fig. 6. There is hardly any change in the prediction even with \( \pm 50\% \) variation about the default value of -0.54. It should be noted that for this Spray G2 condition the chamber is filled with non-condensable gases and \( \alpha \) comprises of both vapor fuel and non-condensable gases. Unlike \( \theta_0 \), \( \alpha \) is a variable that approaches a constant value (\( \leq 1 \)) as the fluid travels through the nozzle and is convected downstream. Hence, along the fluid path line the influence of \( \alpha \) on \( \theta \) diminishes. Therefore, \( \alpha \) is not going to be a dominant factor for the cases of interest in this study.

The parameter \( \psi \), in Eq. 12, incorporates the effect of fuel physical properties in the time-scale estimation, as well as, the essence of local thermody-
Figure 3. Effect of operating and boundary conditions on two-phase flow through Spray G injector: G, G2 and G3. Vapor mass fraction contours are shown through a cut-plane showing 2 of the 8 holes.

Figure 4. Mass fraction and volume fraction contours of vapor iso-octane on two-phase flow through Spray G injector under G2 condition from Table 1.

Figure 5. Effect of changing $\theta_0$ on fuel vapor mass fraction contour for Spray G injector under G2 condition from Table 1.

Figure 6. Effect of changing the exponent of $\alpha$ on fuel vapor mass fraction contour for Spray G injector under G2 condition from Table 1.
namic condition. With increase in temperature for any given fuel, the corresponding saturation pressure increases and so does the value of \( \psi \). This is due to the fact that with increase in \( p_{\text{sat}} \), denominator of \( \psi \) decreases and numerator increases, provided the local pressure remains the same. Exponent of \( \psi \) has been varied from -0.88 to -2.64 with the default value being -1.76. \( \psi \) affects significantly the vapor formation, as depicted in the Fig. 7. \( \psi \) can vary by order of magnitude depending on local pressure values. Consequently, in flashing prone zones, \( \psi \) is going to have significantly high values and it considerably influences the vapor formation. As previously mentioned, Spray G2 condition is supposed to moderately flash and hence, default value of -1.76 as exponent of \( \psi \) should be reasonable for Spray G2 condition.

In the code there is another crucial parameter called \( \psi_{\text{min}} \). For cases where local pressure is equal to the saturation pressure, value of \( \frac{\partial \rho}{\partial \psi} \) in Eq. 11 will blow to infinity, because \( \theta \) becomes zero. To negate this issue, \( \psi_{\text{min}} \) has been used as a thresholding parameter, such that when value of \( \psi \) is below \( \psi_{\text{min}} \) the value of \( \psi_{\text{min}} \) will be used instead of \( \psi \). Typically the value of this parameter is set as \( 10^{-5} \). The values have been varied by few orders, from \( 10^{-3} \) to \( 10^{-6} \). Similar to \( \alpha \), \( \psi \) varies during the run-time and \( \psi_{\text{min}} \) becomes useful only when local pressure is very close to saturation pressure. However, it is imperative from Fig. 8 that prediction of vapor formation is not highly sensitive to \( \psi_{\text{min}} \) values.

On the basis of results presented at this point, it can be inferred that time-scale \( \theta \) is strongly influenced by \( \theta_0 \) and \( \psi \) as well as its exponent. Nevertheless, it is not very clear how \( \theta \) affects the vapor production in order to capture the phase changes under various thermodynamic conditions. A simple mathematical exercise can help in visualizing the qualitative trend of \( \theta \). It is known that local pressure can vary from tens of Pa inside the injector holes to few hundreds of kPa in the near-nozzle region in GDI engines, where flashing usually occurs. Using the mathematical expression of \( \theta \) in Eq. 12, the pattern of variation of \( \theta \) can be investigated for a range of \( \alpha \), and \( \psi \) i.e., local pressure, for a given fuel temperature. For fuel temperatures of 300 K and 360 K, Fig. 9 presents variation of \( \theta \) with \( \alpha \) and local pressure. For a fixed fuel temperature, saturation pressure is fixed and therefore, variation of local pressure over a wide range would encompass the possible range of variation of \( \psi \). The local pressures have been selected such that the qualitative pattern of variation of \( \theta \) becomes apparent. The peak values are occurring typically around the saturation pressure for both 300 K (\( p_{\text{sat}} = 7,320 \) Pa) and 360 K (\( p_{\text{sat}} = 70,900 \) Pa). The peak value of \( \theta \), however, will definitely change depending on selection of local pressure, close to the saturation pressure. The variations of \( \theta \) indicates again, that the local pressure or to be precise, the \( \psi \) parameter is more dominant factor than \( \alpha \) for influencing \( \theta \). For both the cases, it is seen that \( \theta \) decreases when \( \alpha \) approaches unity and is more evident from the magnified view of the 360 K case. Under typical phase change conditions the tendency to produce vapor is supposed to decrease as the gas phase approaches its peak concentration. HRM was originally developed with measured data for mixture of liquid and vapor phases only. Moreover, in computational cells where gaseous phase (vapor fuel + non-condensable gases) is predominant, chance of local vapor quality being close to equilibrium is high. As a result, behavior of \( \theta \) with change in \( \alpha \) appears to be logical.

Since \( \psi \) is more dominant factor, it is important to focus on its effect on \( \theta \). Figure 10 helps in clarifying the role of \( \psi \) in influencing \( \theta \). Close to saturation pressure for a given fuel temperature, \( \psi \) drops down drastically, leading to an abrupt rise in \( \theta \), as seen in Fig. 9. Another noteworthy aspect is that variation of \( \psi \) (and as a consequence the same of \( \theta \)) in Fig. 9 about the saturation pressure is symmetric. In this regard, HRM suffers from the same fundamental in-

Figure 7. Effect of changing the exponent of \( \psi \) on fuel vapor mass fraction contours for Spray G injector under G2 condition from Table 1.
Figure 8. Effect of changing the $\psi_{\min}$ on fuel vapor mass fraction contour for the Spray G injector under G2 condition from Table 1.

Figure 9. Variation of $\theta$ with $\alpha$ and local pressure (kPa) for fuel temperatures of 300 K and 360 K. Zoomed-in view highlights the extent of variations when $\theta$ is approaching the peak values for fuel at 360 K.
Figure 10. Variation of $\psi$ with local pressure (kPa) for fuel temperatures of 300 K and 360 K.

adequacy, as the Linear Rayleigh Equation based cavitation models used in single-fluid (mixture or VOF) and two-fluid (Eulerian-Eulerian) models for studying in-nozzle analyses of diesel injectors [34]. Fundamentally, the process of bubble growth and collapse are not same. Nevertheless, the time-scale required to capture complex bubble collapse is too small to be considered for multi-dimensional simulation involving cluster of bubbles or cavities [34]. As a result effectiveness of HRM, in terms of vapor condensation, for engineering calculations is not inferior compared to the commonly adopted, simple bubble-based approaches.

The next aspect to observe is that the peak values of $\theta$ are similar for both the fuel temperatures. However, for chamber pressures in the order of 50 kPa and higher, $\theta$ values are larger for 360 K compared to 300 K. As explained previously, non-equilibrium phenomenon, i.e., flashing tends to occur at elevated fuel temperatures. Additionally, $\theta$ for 360 K drops down noticeably with further decrease in local pressure. The local pressure inside the GDI holes are in the order of few hundred Pascals. Consequently, the equilibrium time-scales inside the holes are smaller than those outside the injector, in the chamber. As a result, phase changes inside the holes can be considered cavitation as the non-equilibrium effects are low due to small values of $\theta$. Keeping in mind chamber pressure of Spray G2 condition is 53 kPa, the predictions of HRM are very realistic in physical sense for GDI applications. However, for 300 K case, $\theta$ peaks at a very low local pressure, since at low fuel temperature the saturation pressure will also be low. Cavitation, being a near equilibrium phenomenon, should be represented by small equilibrium time-scales, which is clearly not getting satisfied. This is because of the bound at 0 Pa, which basically limits the minimum value of theta for very low local pressures. Therefore, for simulation of cavitation in high pressure diesel injectors with low fuel temperatures, $\theta$ predictions could not be realistic. This indicates predictions of HRM is more meaningful for high fuel temperatures. It can thus be inferred that HRM is a phenomenological model, because it is not derived from the first principles. Therefore, despite having some physical basis, $\theta$ variation might not always be 100 % fundamentally correct. Ideally, a phase change sub-model should consider the effect of local pressure, local temperature and physical properties of the fuel. Inclusion of non-dimensional numbers like Jakob number in the model formulation, thus, could be useful for physically meaningful sub-models. Nevertheless, such endeavors constitute scope for future work.

Eventually it will be interesting to see how the above analysis can be reflected in the typical situations of GDI engine. For Spray G, G2 and G3 conditions, Fig. 11 shows the vapor concentrations, as well as the $\theta$ values in different computational cells in the scatter plots of local pressure versus local temperature. The blue lines correspond to the saturation pressure versus saturation temperature curve for iso-octane. The cells belong to one of the 8 holes and the cells in the corresponding counter-bore are also included. It is evident that cells having local pressure lower than the saturation pressure comprise the major portion of vapor fuel occupying cells. The cells with local pressure close to the saturation pressure have the peak $\theta$ values. Interestingly, few cells with local pressure above the saturation pressure also have some fuel vapor, owing to the mathematical nature of $\theta$, which feeds in to the
Figure 11. Scatter plot of local pressure and local temperature in the different computational cells in one of the 8 holes (including the counterbore) of the Spray G injector under Spray G, G2 and G3 conditions, colored by $\theta$ and vapor mass fraction (blue line corresponds to the saturation pressure versus saturation temperature curve for iso-octane.)
source term equation of fuel vapor species equation. The temperature variations for different conditions are caused by differences in the chamber temperatures. Previous works [22, 23, 32] have indicated that there is backflow of the chamber gas in to the counter-bores. Spray G is exposed to high chamber temperature (573 K), while for Spray G2 and G3 conditions the chambers are at room temperature (293 K). The fuel temperature being 363 K for G and G2, and 413 K for G3, the distributions of the cells in the scatter plots make sense for all the three conditions. In Spray G condition, the fuel heating process occurs entirely at pressure levels above the saturation curve. Cavitation is clearly shown by those cells with isothermal pressure drop and recover. In Spray G2, the low chamber temperature induces some fuel cooling, while simultaneously vapor is being formed because of the pressure being below the saturation level. In Spray G3, the same process occurs, but at higher fuel temperatures, leading to a larger amount of fuel vapor formation.

2 Summary And Concluding Remarks

A numerical parametric study has been carried out for a typical GDI fuel injector. The HRM is shown to be capable of predicting non-flashing yet vaporizing, moderately flashing and intensely flashing scenarios. The set of parametric studies lead to following inferences:

1. $\psi$ varies over several orders of magnitude, both spatially and temporally, in a simulation. Thus, local pressure and physical properties significantly affect HRM predictions, since $\psi$ turned out to be a dominant factor for vapor production.

2. The time scale constant ($\theta_0$) is another dominant factor affecting the vapor distribution considerably. Lower values of $\theta_0$ seems to trigger more flashing, the reason for this was explained. Reasonable values of $\theta_0$ were suggested.

3. Void fraction, i.e., $\alpha$ did not appear to be an influential variable in the HRM prediction. Typically $\alpha$ can only vary from 0 to 1. For Spray G conditions the chamber is initially filled with $\text{N}_2$ gas and hence, $\alpha$ does not vary much during the run-time, especially for flashing conditions.

4. HRM is effective for simulating phase changes for GDI spray applications, despite the fact that it is a phenomenological model with tunable coefficients. This is because, at typical GDI engine chamber pressures, HRM is capable of predicting larger equilibrium time-scales for high fuel temperatures when compared to those for low fuel temperatures. Thus, HRM provides realistic depiction of the flashing scenario.

Overall, HRM even with its caveats is a desirable option for internal combustion engines, whether it is diesel or gasoline fuel. Future modifications in HRM could include effects of Jakob number to further improve its fundamental basis.

Acknowledgment

This research was partially funded by DOE’s Office of Vehicle Technologies, Office of Energy Efficiency and Renewable Energy under Contract No. DE-AC02-06CH11357. The authors wish to thank Gurpreet Singh and Leo Breton at DOE, for their support.

The authors would like to acknowledge Ronald Grover at General Motors and Lyle Pickett at Sandia National Laboratory for providing the nominal geometry of Spray G nozzle. The authors want to express their gratitude to Yanheng Li and Eric Pommaring at Convergent Sciences Inc. for providing better understanding of volume-averaging method for VOF simulations.

The authors are grateful to LCRC computing resources at Argonne National Laboratory.

References


Nomenclature

**CFL** Courant Friedrichs Lewy number

**D** Mass diffusivity ($m^2/s$)

**e** Specific internal energy ($J/kg$)

**h_{lv}** Latent heat of vaporization ($kJ/kg$)

**h_m** Specific enthalpy of $m^{th}$ species ($kJ/kg$)

**Ja** Jakob Number

**k** Turbulent kinetic energy ($m^2/s^2$)

**m_{vap}, m_{liq}** Mass of vapor and liquid phases in a cell ($kg$)

**p** Local pressure ($Pa$)

**P_{ch}, P_{chamber}** Chamber pressure ($kPa$)

**p_{crit}** Critical pressure ($Pa$)

**P_{inj}** Injection pressure ($MPa$)

**P_{sat}, p_{sat}** Saturation pressure ($Pa$)

**R_P** Pressure ratio

**S, S_i, S_e, S_m** Source term in conservation equations

**S_{ij}** Strain rate ($1/s$)

**t** Time ($s$)

**T** Local cell temperature ($K$)

**T_{fuel}** Fuel temperature ($K$)

**T_{sat}** Saturation temperature ($K$)

**u** Local cell velocity ($m/s$)

**u_i** Advecting mean velocity ($m/s$)

**u_j** Advected mean velocity ($m/s$)

**V** Volume of a cell ($m^3$)

**x** Local cell vapor quality

**\bar{x}** Local cell equilibrium quality

**Y_m** Mass fraction of $m^{th}$ species

**Greek**

**\alpha** Void fraction (vapor and non-condensable gases)

**\epsilon** Turbulence dissipation rate ($m^2/s^3$)

**\theta, \theta_0** Equilibrium time-scale and Empirical time-constant ($s$)

**\mu, \mu_t** Dynamic and Turbulent viscosity coefficient ($kg/m.s$)

**\rho, \rho_v, \rho_l, \rho_g** Density of mixture, vapor, liquid and gas ($kg/m^3$)

**\sigma_{ij}** Strain rate tensor ($1/s$)

**\tau_{ij}** Reynold’s stress ($Pa$)