Characterization of Secondary Drops Using Digital In-line Holography

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Abstract

Visualizing the liquid breakup process and quantifying drop morphology, sizes, spatial distribution and three-dimensional (3D) velocity are crucial for characterization and modeling of atomization processes. Digital in-line holography (DIH) is uniquely capable of imaging the breakup geometry and measuring droplet size and 3D velocity distributions. In the present study, a DIH system is configured to record sequential holograms by synchronizing a CCD camera operating in double-exposure mode and a double-pulsed Nd:YAG laser. A recently proposed hybrid method is used to process the holograms to extract the particle information. Particle matching is conducted to extract the 3D three-component velocities of droplets. The DIH system, along with the hybrid method, is applied to characterize the secondary atomization processes due to aerodynamic breakup of drop and drop impact on a thin film. In both applications, the size distribution, 3D spatial distribution and velocity of secondary drops are determined quantitatively. In particular, 3D morphology of the intermediate rim in the bag breakup mode is extracted. Coincidence with the results obtained from phase Doppler anemometry (PDA) measurement is found, demonstrating the accuracy of the measurement by DIH and the hybrid method.

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Introduction

Atomization plays an essential role in a wide variety of scientific and engineering applications including fuel injection, pharmaceutical coatings, spray drying, material processing and etc. The basic goal of an atomization process is to enhance mass transfer and mass dispersal. The sizes of produced droplets substantially determines the rate of evaporation, in which mass is transferred from liquid phase to gas phase. On the other hand, mass dispersal depends on the trajectories or velocities of the drops. Furthermore, knowledge of the three-dimensional (3D) morphology of liquid fragmentation is required for the development and validation of first-principle breakup models. For example, theoretical and analytical models describing the aerodynamic breakup of a single drop are subject to considerable criticism. Previous breakup models based on boundary layer effects may be incorrect [1], and current debate focuses on the relative importance of Kelvin-Helmholtz and Rayleigh-Taylor instabilities [2]. Contributing to the confusion is the lack of 3D, time-resolved experimental characterization of the dynamic process. In summary, the morphology of liquid fragmentation and drop size and velocity distribution are crucial parameters which require experimental quantification for development of models and simulation validation.

Phase Doppler anemometry (PDA) is commonly used to give pointwise measurement of droplet size and velocity, yet it is unable to retrieve spatially resolved information of the droplet field. In addition, the drop is required to have a spherical shape and known refractive index. In two-dimensional (2D) imaging techniques, such as high-speed photography, out-of-focus objects tend to result in erroneous detection, especially when automated image processing is performed.

Digital in-line holography (DIH) has the capability to overcome the limitations of the aforementioned techniques with unique access to 3D information. It has been applied to the characterization of multiphase droplets [3-5], with demonstrated ability to measure the sizes and 3D locations of the droplets in a probing volume without knowing the refractive index of the liquid. Sub-micron accuracy has been reported in determining the size of a particle [6,7], in which the application is limited to dilute particle fields of spherical particles due to the algorithms used to extract the particle information. DIH has also been applied to a denser droplet field generated by a nozzle [8], where the measurement accuracy is only partially validated by comparing mean diameter and velocity with PDA measurements. Combined with tomographic techniques, it is also able to extract the morphology of drops of simple shapes [9]. Consequently, the effectiveness of DIH applied to the characterization of atomization processes remain to be explored, especially when the particle field consists of denser particles of non-spherical shapes and fragments of complex morphologies.

In the current work, DIH, along with the hybrid method, is applied to characterize aerodynamic fragmentation of a drop. The size, 3D spatial distribution, 3D three-component velocities of secondary drops from the bag breakup and 3D morphology of the rim are measured quantitatively. The measurement accuracy is verified by comparison with results from PDA measurement. Further, drop impact on a thin film is characterized using DIH. The size and velocity of secondary drops formed from the jets protruding from the basal crown are measured. The circular symmetry with respect to the crown center of the droplet spatial distribution and velocities proves the validity of the detection.

Methodology

Digital In-line Holography

Shown in Fig. 1(a) is a typical setup of DIH for particle field detection. The laser beam is spatially filtered, expended and collimated before illuminating the particle field. The interference pattern (hologram), between the light diffracted and scattered by the particles (object wave) and the undisturbed part of the same beam (reference wave), is recorded by the CCD sensor and stored in a computer as a digital image \( H(m,n) \). In the reconstruction step, by varying the reconstruction distance \( z_r \), particles at different planes of the reconstruction volume can be brought into focus in the reconstructed images, as shown in Fig. 1(b). Numerical reconstruction is performed using the angular spectrum method, which can be expressed as [10]

\[
E_r(x,y,z_r) = [H(\xi,\eta)R^*(\xi,\eta)] \otimes g(\xi,\eta,z_r),
\]

where

\[
g(\xi,\eta,z_r) = \frac{1}{j\lambda} \exp\left(\frac{jk\sqrt{\xi^2 + \eta^2 + z_r^2}}{\sqrt{\xi^2 + \eta^2 + z_r^2}}\right)
\]

is the Rayleigh-Sommerfeld diffraction kernel. \( E_r(x,y,z_r) \) is the complex amplitude reconstructed at \( z_r \). \( (\xi,\eta) \) and \( (x,y) \) are coordinates in the hologram plane and reconstruction plane, respectively. \( R(\xi,\eta) \) is the complex amplitude of the reference wave, and \( * \) denotes complex conjugate. \( \otimes \) denotes convolution operation. \( \lambda \) and \( k \) are the wavelength and wave number, respectively.
ulation of Eqn. (1) in the Fourier domain can be expressed as

\[ E_r(k, l, z) = \mathcal{F}^{-1}\left\{ \mathcal{F}[H(m, n)] \exp \left( jkz \sqrt{1 - \left( \frac{\Delta M}{N_\xi} \right)^2 - \left( \frac{\Delta N}{N_\eta} \right)^2} \right) \right\}, \]

(3)

where \( R^* \) is omitted, since \( R(m, n) = 1 \) corresponding to a uniform plane wave. \( M \) and \( N \) are number of pixels of the CCD sensor in \( \xi \) and \( \eta \) directions. \( \Delta \xi \) and \( \Delta \eta \) are dimensions of an individual pixel. \( \mathcal{F} \) and \( \mathcal{F}^{-1} \) denote fast Fourier transform and inverse fast Fourier transform, respectively. The reconstruction distance at which a particle is in focus is decided as the depth (z position) of the particle. The transverse position, particle size and shape can be determined from the focused image of the particle.

The Hybrid Method

The hybrid method [11], which features automatic selection of thresholds for segmentation and capability to measure particles of arbitrary shapes, has recently been proposed to extract the particle information. It is applied in the present study to detect the size, shape and 3D location of secondary drops as well as the 3D morphology of liquid fragments. The diameter of a non-spherical particle is determined as that of a spherical particle with an equivalent cross-section area. The detected droplets in sequential holograms are paired using the match probability method [12], and thus the 3D, three-component displacements \( \Delta \vec{x} \) of droplets can also be determined. Finally, the 3D droplet velocity is measured by \( \Delta \vec{v} = \Delta \vec{x} / \Delta t \), where \( \Delta t \) is the time interval between sequential holograms.

Experimental Setup for Aerodynamic Fragmentation of A Drop

Figure 2 shows the experimental configuration for measurement of aerodynamic breakup of a drop. The origin of the coordinate system is located at the center of the circular outlet of an air nozzle. The main air flow direction (+x direction) is perpendicular to the direction of the illuminating beam provided by a double-pulsed, Q-switched Nd:YAG laser (Quantel Evergreen, 532 nm, 5 ns pulse width). The pulse energy is adjusted by varying the time delay between the flashlamp and Q-switch of the laser. To synchronize the drop breakup event with the DIH system, a He-Ne laser and a photodetector are used to generate a trigger signal when the He-Ne beam path is blocked by the falling drop. Hologram pairs are recorded by a CCD camera (IMPERX ICL-B4020, 4008 × 2672 pixels, 9 × 9 μm²/pixel) with \( \Delta t = 62 \mu s \). An ethanol drop, produced by a dispensing tip placed at \((x = 10 \text{ mm}, y = -128 \text{ mm}, z = 0 \text{ mm})\), leaves the tip with near zero velocity and is accelerated due to gravity into the air jet. The total mass flow rate in the nozzle is monitored by a Coriolis mass-flow sensor such that the air-jet velocities can be estimated from previous measurements [13]. Listed in Table 1 are the experimental conditions. The Weber number is about 11, which corresponds to the bag breakup regime [1]. In addition, to verify the accuracy of detection by DIH, a PDA system (Dantec Dynamics) is configured to measure drop size and 2D velocity (in the \( x - y \) plane) with a probing point set at \((x = 150 \text{ mm}, y = 17 \text{ mm}, z = 0 \text{ mm})\), where the
The non-dimensional time $\tau$ is defined as $\tau = u_0 t / d_0$. The thickness of the film $\delta = h / d_0$ is also measured from high-speed imaging. The initial drop diameter and impact velocity are measured from high-speed imaging. The initial drop has fully broken into secondary drops of spherical shapes.

### Table 2. Experimental conditions for drop impact on a thin film

<table>
<thead>
<tr>
<th>Weber number $We = \rho u_0^2 d_0 / \sigma$</th>
<th>Initial diameter $d_0$ (mm)</th>
<th>Impact velocity $u_0$ (m/s)</th>
<th>Dimensionless film thickness, $\delta = h / d_0$</th>
<th>Dimensionless time $\tau = u_0 t / d_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>722</td>
<td>3.18</td>
<td>4.05</td>
<td>0.739</td>
<td>1.7</td>
</tr>
</tbody>
</table>

**Aerodynamic Fragmentation of A Drop**

Figure 4 shows morphological development of the drop in the course of bag breakup process and the corresponding holograms. The drop first develops to a pan-like shape then to a thin hollow bag attached to a thicker toroidal rim. The bag further breaks up into a large number of small drops, followed by the disintegration of the rim. The hybrid method is applied to the highlighted hologram in Fig. 4, followed by particle matching for velocity extraction. The measured rim morphology, drop size and velocity are shown in Fig. 5. The 3D geometry of the rim is extracted assuming the rim is composed of differential tube segments with circular cross-section. Edge pixels of each segment are recognized from the background due to their high values in the maximum Tenengrad map [11], as shown in the insert of Fig. 5. Further, each edge pixel is associated with a 3D coordinate (according to the depth map). Accordingly, the diameter, center location and orientation of each segment are determined from the edge pixels. Linking differential segments together produces the rim. The volume of the rim is estimated directly from the 3D geometry, which corresponds 90.87% of the volume of the initial drop. The volume of all detected secondary drops is found to be 9.49% that of the initial drop. Previous estimation of the volume ratio of the rim is 56% [14], which is significantly different from the measurement result here. The dominant velocity component is in $+x$ direction due to the aerodynamic drag. Disintegration of the bag casts the droplets outward radially in the $y-z$ plane, so the droplets also gain velocities in the $y$ and $z$ directions.

In the setup in Fig. 2, a unit magnification is achieved, and the lower limit of the detectable diameter is determined by the pixel size of the CCD. Here, the minimum detectable diameter is set to 30 $\mu$m ($\sim$3 pixels). However, PDA has a much lower limit on detectable diameters. Therefore, to make a valid comparison with PDA measurement, the DIH system is modified by placing a lens between the object and the CCD to introduce a magnification, so that
Figure 4. Illustration of the bag breakup process and the corresponding holograms. The scale bar represents 4 mm. Note, illustrating holograms are cropped from full-size holograms.

Figure 5. 3D representation of the measured morphology of the rim, drop spatial distribution, size and velocity. Insert: cropped maximum Tenengrad map. D: diameter.

Particles smaller than 30$\mu$m can be effectively resolved. The distance between the camera and the lens is determined via calibration as 889.7 mm with a standard deviation of 1.5 mm. Finally, an approximate magnification of 3.5 is achieved. 11508 drops are measured by PDA, and 4105 are detected from 18 holograms using DIH. The field of view of DIH is located where the bag has broken into droplets, yet the rim remains intact. Therefore, DIH measures only the small droplets produced by breakup of the bag, while the downstream PDA measurement may include larger drops formed from breakup of the rim. Nevertheless, as shown in Fig. 6, agreement between DIH and PDA for the size distribution is quite good. This is likely because the number of drops produced from breakup of the bag is significantly greater than the number produced from breakup of the rim. Furthermore, Table 3 compares the measured mean droplet diameters and velocities, where the definitions of the mean diameters can be found in [1]. The discrepancies in mean velocities can be attributed to differences in measurement location, noting that drops measured by the downstream PDA have been further accelerated by aerodynamic drag. These effects can be estimated to first order by assuming the forces acting on the droplets are the gravity force and steady-state aerodynamic drag by a uniform flow field ($u_x = 9$ m/s; $u_y = u_z = 0$). With this, the droplets measured by DIH are numerically propagated to the plane of the PDA measurements ($x = 150$ mm) resulting in an estimated mean $x$ velocity of 8.64 m/s.

Table 3. Comparison of Measured Mean Diameters ($\mu$m) and Velocities (m/s)

<table>
<thead>
<tr>
<th></th>
<th>$d_{10}$</th>
<th>$d_{30}$</th>
<th>$d_{32}$</th>
<th>$u_x$</th>
<th>$u_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PDA</td>
<td>48.97</td>
<td>78.73</td>
<td>123.87</td>
<td>7.65</td>
<td>0.44</td>
</tr>
<tr>
<td>DIH</td>
<td>49.42</td>
<td>76.45</td>
<td>115.96</td>
<td>4.54</td>
<td>0.66</td>
</tr>
</tbody>
</table>

Drop Impact on A Thin Film

Shown in Figs. 7(a) and (b) are a sample image from high-speed imaging and the corresponding hologram. The reconstruction of the hologram
Figure 7. High-speed image (a), recorded hologram (b) of drop impact at $We = 722, \tau = 1.7$ and cropped reconstruction of the hologram (c). The scale bar is 4 mm.

Figure 8. Detected secondary droplets projected to the $x - y$ plane and the $x - z$ plane. $D$: diameter.

Figure 9. Measured size distribution of secondary droplets from drop impact.

Conclusion

The secondary atomization processes produced by drop impact on a thin film and aerodynamic breakup a drop are characterized using DIH and the hybrid method. The size, 3D spatial distribution and velocity of droplets as well as the complex non-spherical morphology of the rim in bag breakup mode are determined quantitatively. The measurement accuracy is verified by comparison with results from PDA measurements and analytical interpretations of the results. The potential of DIH to extract highly non-spherical, 3D morphologies is also demonstrated. Finally, DIH, along with the hybrid method, is proven to be an excellent tool for the characterization of secondary atomization processes.

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Nomenclature

\[ \begin{align*}
\text{d} & \quad \text{diameter} \\
E & \quad \text{complex amplitude} \\
F & \quad \text{fast Fourier transform} \\
g & \quad \text{Rayleigh-Sommerfeld diffraction kernel} \\
h & \quad \text{liquid film thickness} \\
H & \quad \text{hologram} \\
k & \quad \text{wave number} \\
M & \quad \text{number of pixel in horizontal direction} \\
N & \quad \text{number of pixel in vertical direction} \\
R & \quad \text{reference wave} \\
t & \quad \text{time} \\
u & \quad \text{velocity} \\
We & \quad \text{Weber number} \\
\lambda & \quad \text{wavelength} \\
\Delta \xi & \quad \text{pixel dimension in horizontal direction} \\
\Delta \eta & \quad \text{pixel dimension in vertical direction} \\
\tau & \quad \text{dimensionless time} \\
\rho & \quad \text{density} \\
\sigma & \quad \text{surface tension} \\
\delta & \quad \text{dimensionless film thickness} \\
\end{align*} \]

Subscripts

\[ \begin{align*}
0 & \quad \text{initial} \\
a & \quad \text{air} \\
l & \quad \text{liquid} \\
r & \quad \text{reconstruction} \\
rel & \quad \text{gas-liquid relative} \\
\end{align*} \]

References


