Validation of Cavitation Simulations in Submerged Nozzles

Daniel J. Duke*, Michele Battistoni1,2, Andrew B. Swantek1, Nicholas Sovis1, Alan L. Kastengren3, Christopher F. Powell1, Sibendu Som1 and David P. Schmidt4

1 Energy Systems Division
Argonne National Laboratory
Lemont, IL 60439 USA

2 University of Perugia, Italy

3 X-ray Science Division
Argonne National Laboratory
Lemont, IL 60439 USA

4 Department of Mechanical and Industrial Engineering
University of Massachusetts-Amherst
Amherst, MA USA

Abstract

Recent experimental and numerical studies of nozzle cavitation have revealed that dissolved non-condensable gases can play a substantial role in determining both the degree of cavitation and the morphology of the vapor field. These phenomena are currently not well understood, and are challenging to simulate. A wide variety of physical sub-models are used to model cavitation, yet the influence of non-condensable gases on the accuracy of these models has not yet been investigated, owing to the lack of available experimental data. To address this problem, new x-ray fluorescence measurements were performed at the 7-BM beamline of the Advanced Photon Source (APS) at Argonne National Laboratory. The fluorescence measurement was able to separate the contribution of dissolved gas and cavitation on the total line of sight void fraction. We consider the simplified case of a submerged cavitating nozzle with a sharp inlet and fixed diameter of 500 micron. Cavitation of a gasoline fuel surrogate was simulated with a compressible homogeneous relaxation model (HRM). Two implementations are considered, using Large Eddy Simulation to model turbulence in three-dimensional geometries. A range of conditions are considered, covering both incipient and strongly cavitating conditions, with and without dissolved gas in the fuel. A quantitative comparison between the simulations and line-of-sight x-ray measurements is made by projecting the volume fractions of cavitation vapor and non-condensable gas onto a plane. For the first time, both the concentration of dissolved gas in the fuel and the void fraction due to cavitation can be simultaneously validated.

* corresponding author: dduke@anl.gov
Introduction

The study of cavitation in small nozzles is a problem of particular interest for its application to direct injection systems for internal combustion [1]. Cavitation is particularly important in governing the internal flow of diesel injector nozzles [2,3]. The small diameters and high pressures cause extreme pressure gradients near the nozzle inlet, which promote cavitation inception at the nozzle wall. Progressive increases in injection pressure reduce the effectiveness of geometric modifications such as hydro-grinding and hole taper. Cavitation suppression is becoming increasingly difficult in direct injection applications [4].

The general principles of cavitation bubble formation and collapse have been studied for some time, and are well-established [5,6]. However, the gross structure of cavitating nozzle flow is still not particularly well-understood. Optical injector experiments have revealed that clouds of individual cavitation bubbles can merge and form various structures such as sheets and strings [7]. These structures involve complex hydrodynamic and thermodynamic interactions, posing unique challenges for both experiments and simulations.

Recent work has revealed the presence of another complicating factor in cavitating nozzle flows; the influence of dissolved non-condensible gases [8,9]. Nitrogen and oxygen dissolve readily in hydrocarbon fuels [10] and are in equilibrium with partial pressures orders of magnitude higher than the vapor pressure of the fuel itself. However, the time scales associated with the gas dynamics are also orders of magnitude slower than cavitation time scales; the end result is a complex interplay between both true cavitation and non-condensible gas precipitation and dissolution [11].

Since real-world injector flows will always contain some quantity of dissolved gas, one would expect to observe the nucleation of dissolved gas bubbles wherever cavitation nucleation occurs, although the effect may be observed further downstream than cavitation due to the longer time scales [8,9]. However, making an independent experimental measurement of the local quantities of non-condensible gases and true cavitation vapor is particularly challenging.

Numerical simulation of cavitating nozzle flows has been investigated for some time [12]. A large variety of physical sub-models have been investigated [13-15]. In particular, homogeneous relaxation models have shown good results simulating cavitation and flashing of multi-component fuels [4,16,17]. The inclusion of transport equations for non-condensible gases has also been explored [4], however there is presently a lack of experimental validation data for these predictions. Simulation validation data are typically in the form of side-by-side comparison with imaging in optical nozzles, or total vapor fraction only [8,18,19].

With respect to experimental validation, recent developments in optical fuel injection experiments now allow images to be taken of real-size models of injectors. This is a significant advance, since cavitation does not scale with other hydrodynamic parameters [20,21]. However, the scattering of visible light at gas-liquid boundaries and refraction through the nozzle walls makes quantitative analysis of the degree of cavitation challenging.

In order to overcome these limitations, one can turn to x-ray diagnostics [8,22]. X-rays scatter only weakly, making direct line of sight measurements more feasible. X-ray phase contrast imaging using a synchrotron source has been demonstrated effectively for cavitating wedges and nozzles [4,23,24]. Furthermore, x-ray computed tomography has also been applied to scaled-up cavitating nozzle flow [25]. Recently, quantitative measurements of cavitation in submerged plastic nozzles have been demonstrated using time-resolved x-ray radiography [8,17]. X-ray diagnostics provide a useful quantitative benchmark for numerical simulations, since line of sight projections of the vapor fraction can easily be recovered from three-dimensional simulations [9,26]. However, these techniques are not adapted to separating the effects of non-condensible gases and true cavitation; only the total vapor fraction has been measured.

To address this problem, we consider the application of x-ray fluorescence. Using a high-flux synchrotron source, fluorescing tracers can be used to tag a fluid flow, allowing multiple species to be separately measured [27]. Since the fluorescence photons are x-rays, only a small fraction are scattered by or reabsorbed in the working fluid and nozzle wall. With some corrections for these effects, simultaneous species concentration measurements can be made along a line of sight [28].

In this paper, we model the flow of a gasoline surrogate through a canonical submerged nozzle 500μm in diameter. A homogeneous relaxation model (HRM) is considered in several implementations, allowing for compressibility of the fluid. The transport of non-condensible gas around the cavitation zones is investigated and the results are validated against novel x-ray fluorescence measurements of cavitation vapor and dissolved gas concentration.
Experimental Method

The nozzle geometry considered in this study is shown in Fig. 1. The nominal diameter is \( D = 500 \mu m \) and the inlet has a 50 \( \mu m \) 45-degree bevel, which was chosen for practical machining purposes. The nozzle body is made from polyetheretherketone, a high-hardness plastic that is able to tolerate the radiation damage due to prolonged x-ray exposure. The nozzle length is \( L = 3.05 \text{mm} \), giving \( L/D = 6.1 \). The inlet and outlet sections have plenum spaces 2.5 mm in diameter. The boundary conditions are described by pressures and temperatures \( P_1, T_1, P_2 \) and \( T_2 \), and volume flow rate \( \dot{V} \). From these boundary conditions we derive the Reynolds and Cavitation numbers

\[
Re = \frac{4 \dot{V}}{\nu \pi D} \\
\sigma = \frac{P_2 - P_0}{P_1 - P_2}
\]

where \( \nu = 1.5 \text{ cSt} \) is the fluid kinematic viscosity and \( R_t = 4.6 \text{ torr} \) is the vapor pressure for the commercial gasoline surrogate used in the experiments.

Liquid is delivered under constant pressure and flow rate to the submerged nozzle using a gas-driven piston accumulator system which has been described in detail in previous work [8,26]. The nozzle is held in a compression fixture in a vertical orientation (fluid flows from top to bottom) and is scanned through the fixed x-ray beam.

X-ray radiography is limited to measuring only the total density of a fluid along the beam path [8,17,22,26]. To measure both cavitation vapor and dissolved gas concentration, we use x-ray fluorescence. Like optical fluorescence, the number of detected photons can be correlated to the number density of a particular tracer in the beam, with some corrections [27].

\[ \text{Do} = \sigma 2.50 \text{ mm} \]

\[ D = \sigma 0.50 \text{ mm} \]

\[ P_1, T_1 \]

\[ P_2, T_2 \]

\[ \text{flow} \]

\[ 0.05 \text{ mm} \]

\[ L = 3.05 \text{ mm} \]

**Figure 1.** Cross-section of nozzle geometry.

For the purposes of these experiments, the liquid is doped with a 400 ppm concentration of tetrabromo-methane (CBr\(_4\)), which dissolves readily in hydrocarbons. Bromine is an ideal tracer, since it has a high fluorescence yield and high K-shell fluorescence energies (11.88 - 11.92 keV). In experiments where dissolved gases are considered, the air dissolved in the fuel is removed using a vacuum pump and krypton is substituted. It also has high K-shell fluorescence energies (12.60 - 12.65 keV), which are distinct and easily resolved from those produced by Br. The high energy allows approximately 50% of the fluorescence photons to survive to the detector without reabsorption through the nozzle wall material and air [30]. Krypton is slightly more soluble than air in hydrocarbons and has a higher molar mass, but the equilibrium dissolved gas concentration at atmospheric conditions is at the same order of magnitude [10,31].

The x-ray measurements were performed at the 7-BM beamline of the APS [32]. The mean energy of the monochromatic beam was set at 14.7 keV, 1.0% full width at half maximum band-pass energy. The beam was focused to a spot approximately 5 \( \times \) 6 \( \mu m \) in size using a pair of Rh-coated Kirkpatrick-Baez mirrors. The beam focus was aligned with the center of the nozzle. The divergence of the beam is small compared to the diameter (less than 3 mrad), so its cross-section remains relatively constant across the nozzle diameter. The beam intensity was monitored upstream of the x-ray beam.

**Figure 2.** Planar cut through nozzle, illustrating the x-ray fluorescence setup. From this view, the flow is traveling into the page.
experiment \(I_0\), and the transmitted beam \(I_1\) was measured with a PIN diode. The nozzle was traversed through the fixed x-ray beam to build up a raster-scan set of measurements.

The x-ray fluorescence setup was similar to the radiography setup described in previous work [8,26], but with the addition of photon-counting silicon drift detector placed at 100 degrees to the incident beam, as shown in the cross-section in Figure 2. The coordinate system was aligned with \(x\) transverse and \(y\) axial, and \(+z\) in the incident x-ray beam direction. The fluorescence detector is at an angle \(\theta\) relative to the beam in the \(x\)-\(z\) plane. It is a paralyzable detector, only receptive to detecting photons for a fraction of the total interrogation time, denoted \(t_{\text{live}}/\Delta t\). The detector software estimates this value, and a post-correction is applied to handle the nonlinearity of the detector when the count rate is high [33]. The detector returns an energy spectrum \(I(E)\) in arbitrary units proportional to the photon count rate.

For each tagged fluorescent species \(k\), the density projected along the beam path \((\text{g/cm}^2)\) is calculated as

\[
m_k = \int \rho_k(x, y, z) \, dz = \frac{c_k f_s(x, y)}{f_s(x)} \frac{t_{\text{live}}}{I_0} \frac{E_{k,\text{max}}}{E_{k,\text{min}}} \int_{E_{k,\text{min}}}^{E_{k,\text{max}}} I(E, x, y) - I_s(E, x, y, \theta) \, dE
\]

(3)

where \(I_s\) is the background spectrum, which accounts for the tail of the Compton scattering curve into the energy range \((E_{k,\text{min}}, E_{k,\text{max}})\). This energy range covers the region of interest in the energy spectrum for species \(k\). Normalizing factors include the incoming beam energy \(I_0\) and the attenuation of the incident beam in the sample \(f_s(x)\), which is calculated using the Lambert-Beer law.

The unknown \(c_k\) in Equation (3) is a proportionality constant which depends on the solid angle, fluorescence efficiency and number density. \(f_s\) is a signal trapping correction which depends on the fraction of photons transmitted through the nozzle wall, and through the fluid inside the nozzle on their way from the line source created by the beam and the detector (see Fig. 2). Whilst the other quantities are constants, \(f_s\) depends on the density field.

Solving eqn. (3) for the projected densities \(m_k\) and the unknowns requires iteration, since the signal trapping coefficient and the density fields for each species depend on each other. The solution procedure is described as follows.

As inputs, we consider sets of fluorescence measurements projected at angle \(\theta\) from the incident beam for several species \(k \in [0, M]\), the known constants, and the radiography measurements of total projected mass for all species which gives \(f_k\). Given an axisymmetric geometry, Abel inversion is used to estimate the spatial distribution (in a plane) of each species. This provides an approximation of the density distribution, including both the fluid and the nozzle wall. In the first iterative step, we assume the signal trapping coefficients \(f_k = 1\). The density fields are then projected in the beam direction at each position in the raster scan grid. The known mass attenuation coefficients of the dopants and their solvents are used to develop a set of linear equations describing the total attenuation of the incident beam as a function of the densities of each species and its solvent, requiring that the projected mass from all tracer species and solvents add up to the total absorption measured by radiography \(a_r = I_r(x, y)/I_0(x, y)\), allowing for absorption in the nozzle wall and air gaps. The total absorption problem is described by the equation

\[
\arg\min_{q} \|Aq - a_r\|^2
\]

(4)

where \(\| \cdot \|\) represents the 2-norm, \(A\) is a matrix of non-dimensional absorption values \(a_{k,x}\) for each species \(k\) at any raster-scan location \(x\);

\[
A = \begin{bmatrix}
-1 & a_{w,x} & \vdots & a_{k_0, x} & \cdots & a_{k_M, x} \\
-1 & \vdots & \ddots & \vdots & \ddots & \vdots \\
-1 & a_{w,x} & \vdots & a_{k_0, x} & \cdots & a_{k_M, x}
\end{bmatrix}
\]

(5)

where the absorptions are described by the Lambert-Beer law;

\[
a_{k,x} = \exp\left(-\mu_k \int \rho_k(x, z) \, dz\right)
\]

(6)

and similarly for the total absorption in the radiography \(a_r\) and the absorption in the nozzle walls \(a_{nx}\). \(\mu_k\) are the known mass attenuation coefficients of each species at the incident beam energy. The vector \(x\) represents the plane of the orthogonal coordinates \((x, y)\) and \(+z\) is aligned with the beam. In Eq. 4, \(q\) is a vector of proportionality constants for each fluorescent species, along with a constant \(c_w\) accounting for beam absorption in the nozzle wall, and a bias \(c_0\);

\[
q = [c_0 \ c_w \ c_{k_0} \ \cdots \ c_{k_M}]^T.
\]

(7)

The problem of Eq. 4 is solved using the raster-scan points \(x_n\) for \(n \in [0, N]\). Given that \(N \gg M\), the system is over-constrained and is solved using a least-squares approach, requiring that \(q\) have positive coefficients.
Once the coefficients are estimated, the density fields are quantitatively calibrated in units of \( \text{cm}^3/\text{g} \), and the signal trapping coefficients can be estimated using a two-dimensional ray projection from the line source of the \( x \)-ray beam to the detector, using the mass attenuation coefficients at the appropriate emission energies. The volume projections for the Abel inversion can then be updated with the new density estimates. The updated density fields can then be used to improve the coefficient estimation, and the process is iterated with under-relaxation until all the coefficients are acceptably converged within the uncertainty of the original data.

In this paper, we define the local mass fractions of vapor \( x \) with respect to the fuel mass (vapor fuel + liquid fuel), and the local mass fraction of non-condensible gases \( Y_g \) with respect to the total mass in any control volume;

\[
x = M_v / (M_v + M_t)
\]

(8)

\[
Y_g = M_g / M_{tot}
\]

(9)

\[
V_{tot} = V_v + V_l + V_g
\]

(10)

\[
M_{tot} / \rho = M_v / \rho_v + M_t / \rho_t + M_g / \rho_g
\]

(11)

where subscript \( v \) represents cavitation vapor, \( l \) represents liquid phase, and \( g \) represents non-condensible gas. From these mass fractions we define local volume fractions for cavitation vapor (\( \alpha_v^* \) with respect to the fuel volume) and non-condensible gases (\( \alpha_g \) with respect to the total volume) as

\[
\alpha_v^* = V_v / (V_v + V_l)
\]

(12)

\[
\alpha_g = V_g / V_{tot} = Y_g (\rho / \rho_g)
\]

(13)

For consistency, we also define mass and volume vapor fractions which are normalized by the total mass;

\[
Y_v = M_v / M_{tot} = x (1 - Y_g)
\]

(14)

\[
\alpha_v = V_v / V_{tot} = \alpha_v^* (1 - \alpha_g)
\]

(15)

The corrected bromine density projections are proportional to the total projected liquid volume fraction

\[
m_{Br} = \int (1 - \alpha_v - \alpha_g) \, dz
\]

(16)

Furthermore, the krypton density projections provide \( m_{Kr} = \int Y_g \rho \, dz \). Knowing the total path length through the nozzle \( \delta z \) and the density of the noncondensible gases \( \rho_g \) downstream of the pressure drop, the projected values of \( \alpha_v \) and \( \alpha_g \) can both be recovered over the raster-scan pattern, with the projected cavitation vapor alone being given by

\[
m_{cav} \approx \int \alpha_v \, dz = \delta z - m_{Br} - \rho_g \delta m_{Kr}.
\]

(17)

Both \( m_{cav} \) and \( m_{Kr} \) have units of length, but can also be expressed as non-dimensional projected fractions by normalizing by the path length through the nozzle, \( \delta z \).

Error propagation calculations suggest that the proportional uncertainty in \( m_{cav} \) is approximately \( \pm 1.1\% \), and \( \pm 2.2\% \) for \( m_{Kr} \). The non-condensible gas density is lower relative to the background level, and the higher energy is closer to the Compton scattering peak, introducing more noise. By way of comparison, radiography measurements of the same flow have an uncertainty of approximately 2% of a typical maximum value for the total projected vapor fraction.

**Numerical Method**

Numerical simulations have been undertaken using two codes based on different platforms, namely OpenFOAM and CONVERGE. Here, the main objective of the modeling work is to provide a robust support for the physical interpretation of the results, rather than to compare and validate different implementations. Both models use the assumption that the multi-phase system is represented by a single-fluid mixture and employ the homogeneous relaxation model (HRM) to describe the occurrence of phase change due to cavitation. The presence of non-condensible gas is also accounted for in the models. The two-phase system comprises three components, i.e. liquid fuel, fuel vapor and non-condensible gas. All the components are compressible, both in the liquid and gaseous state.

With the assumption of local homogeneity, the conservation equations for mass and momentum read as

\[
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_j}{\partial x_j} = 0
\]

(18)

\[
\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial \sigma_{ij}}{\partial x_j}
\]

(19)

where \( \rho \) is the mixture density and \( \sigma_{ij} \) is the stress tensor which includes both viscous and turbulent stresses. The energy equation is also included, even if it is of marginal importance in room temperature cavitation experiments. The formulation used in CONVERGE is written in terms of conservation of sensible internal energy \( e \) and accounts for the effects of viscous and turbulent dissipation, thermal conductivity, and species enthalpy diffusion.
\[
\frac{\partial \rho e}{\partial t} + \frac{\partial \rho \textbf{e} \textbf{u}_j}{\partial x_j} = -\rho \frac{\partial u_j}{\partial x_j} + \sigma \frac{\partial u_j}{\partial x_j} + \frac{\partial}{\partial x_j} \left( \lambda \frac{\partial T}{\partial x_j} \right) + \frac{\partial}{\partial x_j} \left( \rho D \textbf{h} \frac{\partial Y_k}{\partial x_j} \right) + s_e \tag{20}
\]

The energy conservation in HRM Foam is written in terms of enthalpy \( h \) as in Eqn. 21. The last term represents energy dissipation through turbulence;

\[
\frac{\partial \rho h}{\partial t} + \frac{\partial \rho h \textbf{u}_j}{\partial x_j} = \frac{\partial \rho}{\partial t} + \frac{\partial h}{\partial t} \frac{\partial \textbf{u}_j}{\partial x_j} + \frac{\partial}{\partial x_j} \left( \rho \frac{\partial \textbf{u}_j}{\partial x_j} \right) + \Phi \tag{21}
\]

The mixture has a variable density given by the weighted average of the single components densities

\[
\rho = \sum (\alpha_i \rho_i) \tag{22}
\]

where \( \alpha_i \) is the volume fraction of each component. Volume fractions \( \alpha_i \) and mass fractions \( Y_i \) are related through

\[
\rho \alpha_i = Y_i \rho \tag{23}
\]

Using equations (11), (22) and (23), the global void fraction \( \alpha_{\text{glob}} \) takes the form

\[
\alpha_{\text{glob}} = \frac{Y_v / \rho_v + Y_s / \rho_s}{\rho} \tag{24}
\]

This definition of global void fraction sums up the voids generated by phase change (true cavitation) and by the expansion of non-condensable gases.

In general, mass transfer due to phase change can occur at the liquid/gas interface. In a multi-component mixture, it can occur between the liquid fuel and its vapor, or it can potentially occur when gas is released from or absorbed into the liquid solution. Different species are involved in the two cases and different mechanisms apply. Currently, both codes implement the HRM model for the occurrence of liquid-vapor phase change. In addition, a transport equation for the evolution of the non-condensable gas is solved in both codes.

In the OpenFOAM implementation, no mass transfer is assumed to occur between the liquid and the non-condensable gas. This means that the amount of air assumed in the mixture is already in a gaseous form and can only be transported, expand or be compressed, but adsorption or release from the liquid phase are ignored. In more detail, the HRM cavitation model is based on the work of Neroorkar et al. [16], Schmidt et al. [38] and Downar-Zapolski [34]. The model assumes a simple first order rate equation for the evolution of the instantaneous vapor quality \( x \) towards its equilibrium value \( \bar{x} \) over an empirical time-scale \( \Theta \):

\[
\frac{dx}{dt} = \frac{\bar{x} - x}{\Theta} \tag{25}
\]

where the instantaneous non-equilibrium vapor quality \( x \) is defined in accordance with equation (8). The equilibrium vapor quality \( \bar{x} \) is a function of the thermodynamic properties at the local pressure, i.e., \( \bar{x}(\rho, h) \) is expressed by the following equation with bounds at zero and unity:

\[
\bar{x} = \frac{h - h_{\text{sat}}}{h_{\text{v,sat}} - h_{\text{sat}}} \tag{26}
\]

where \( h \) is the actual fluid enthalpy of the liquid and vapor mixture, excluding air content, and \( h_{\text{v,sat}} \) and \( h_{\text{sat}} \) refer to saturated liquid and vapor, respectively. The time-scale depends on the local void fraction and thermodynamic conditions [34];

\[
\Theta = 3.84 \cdot 10^{-7} \alpha^{-0.54} \phi^{-1.76} \text{ (s)} \tag{27}
\]

where subscripts ‘sat’ and ‘crit’ denote saturation and critical pressures, respectively. Correlation provided by combining equations (27-28) has been determined in experiments of flashing flows of water in pipes with upstream pressures of in excess of 10 bar. In the absence of specific values for other liquids and conditions, it has been applied in the present form in this study for representing the gasoline fuel.

The OpenFOAM [35] implementation of HRM uses NIST lookup tables to determine fluid properties [36] using a four-component gasoline model. The pressure equation is solved using a pressure-implicit split-operator (PISO) predictor-corrector algorithm [38]. All quantities are solved to second-order. Further details can be found in prior work [4,16,17,26].

The CONVERGE implementation also uses the PISO algorithm for the pressure equation. Other transport equations, such as the energy, species, and turbulence, are solved after the momentum predictor and the first corrector have been completed. Spatial discretization is second-order accurate. Previous studies using this algorithm can be found in [9,39,41].

The simulations are fully three-dimensional and large eddy simulation (LES) has been used to model turbulence in both codes. In OpenFOAM, turbulence is modeled using a single \( k \)-equation large eddy simula-
tion [37]. In CONVERGE, the dynamic structure sub-grid-scale model is used, which employs a transport equation for the sgs-k as well, but does not use turbulent viscosity to model the sgs-stress tensor [40].

The gridding methods used in the two codes are slightly different, but have the commonalities of cell size and shape in the nozzle region. Hexahedral elements are used, with 160 cells uniformly spaced across the nozzle throat, giving a minimum cell width of 3.125µm. Large plenum spaces are included to reduce the effects of pressure wave reflections; 5D upstream and 20D downstream of the nozzle. An unstructured mesh is used in OpenFOAM next to the wall boundaries and to manage the transition between different embedding levels in the plenums. A Cartesian cut-cell method which preserves orthogonality is used in CONVERGE. Convergence of cavitation and gas volume fractions have been investigated using Richardson extrapolation with two levels of successively refined meshes. The solver was run on a cluster facility until the mass flux reached steady-state conditions.

The convergence of the solution for this class of problems has been discussed in previous work [26]. Grid convergence statistics are difficult to estimate from a single LES; we estimate the convergence using Richardson Extrapolation following Roache’s method [42]. In this study, for the finest mesh (3.125 µm cell size) we obtain grid convergence factors GCI=0.6% for the time-average values of α.

Results and Discussion

The experimental conditions listed in Table 1 were simulated. Case ‘A’ is the reference for the parametric study. Cases ‘A’ and ‘B’ have matched cavitation number but varying Reynolds number, ‘A’ and ‘C’ have matched Reynolds number and varying cavitation number, and ‘A’ and ‘G’ have matched cavitation and Reynolds number, but ‘G’ introduces a finite amount of dissolved krypton into the incoming flow.

Time snapshots of the simulation data were projected at 90 degrees to the streamwise direction to match the projected view obtained from the x-ray experiments. The projection of the unstructured mesh onto a regular grid was computed using Delaunay triangulation [43], with a KD-Tree used to determine the proximity of the cell centers to the wall boundary [44]. For the Cartesian grid case, exploiting the cell uniformity along the line-of-sight, the projection onto a 2D mesh is straightforward and does not require interpolation [9]. In this study, we consider projections of the instantaneous LES fields in order to observe the effect of turbulent structure on the cavitation.

The results for Condition ‘A’ are shown in Figure 3. Figures 3(a,c,d) are expressed in terms of projected cavitation vapor as described by Eqn. 17. We note that the experimental data (Fig. 3a-b) predict a substantially larger amount of cavitation than the OpenFOAM simulation (Fig. 3c), and this trend is repeated for all the conditions studied. Vapor production is observed near both the nozzle inlet edge, and the inner bevel edge, with most of the vapor being located near the nozzle walls. The CONVERGE simulation (Fig. 3d) shows good agreement with the experimental data in terms of the peak values and distribution of projected vapor. The projected instantaneous LES field also shows several flow structures, with cavitation packets that tend to detach from the wall in the second half of the orifice. Cavitation persists in case ‘A’ for the full length of the nozzle, however the nozzle outlet was not visible in the experiments due to an o-ring blocking the field of view.

The highest values for projected vapor occur approximately 100 µm from the nozzle wall and are well matched between simulations and experiments. Figs. 3a-b suggest that the cavitation bubble may be detached from the wall, and a re-entrant jet may be present [1]. Conversely, the simulation results (Fig. 3c) suggest that the cavitation should remain attached to the wall. The simulation results in Fig. 3d, although not time averaged, show the incipient re-entrant behavior of vapor clouds towards the nozzle centerline.

Figures 4 and 5 show the results for case ‘B’ and case ‘C’ respectively, where the fuel remains degassed but the cavitation and Reynolds numbers are varied. The experimental data show that decreasing Re causes a more rapid cavitation collapse with similar total quanti-

<table>
<thead>
<tr>
<th>Case</th>
<th>Inlet pressure $P_1$ (bar abs.)</th>
<th>Outlet pressure $P_2$ (bar abs.)</th>
<th>Flow Rate (L/hr)</th>
<th>Cavitation Number</th>
<th>Reynolds Number</th>
<th>Dissolved gas concentration (mass)</th>
</tr>
</thead>
<tbody>
<tr>
<td>'A'</td>
<td>10.00</td>
<td>1.00</td>
<td>21.9 ± 1.1</td>
<td>0.11</td>
<td>1.5 x 10^4</td>
<td>0</td>
</tr>
<tr>
<td>'B'</td>
<td>1.11</td>
<td>0.11</td>
<td>7.3 ± 0.3</td>
<td>0.11</td>
<td>4.9 x 10^2</td>
<td>0</td>
</tr>
<tr>
<td>'C'</td>
<td>12.00</td>
<td>3.00</td>
<td>23.7 ± 1.3</td>
<td>0.33</td>
<td>1.5 x 10^4</td>
<td>0</td>
</tr>
<tr>
<td>'G'</td>
<td>10.00</td>
<td>1.00</td>
<td>21.5 ± 0.6</td>
<td>0.11</td>
<td>1.5 x 10^4</td>
<td>1.039 x 10^-4</td>
</tr>
</tbody>
</table>

Table 1. Conditions from x-ray experiments, simulated in this study.
ty of vapor (Fig. 4a) and increasing $\sigma$ causes an increase in the total amount of vapor (Fig. 5a). In both cases, the OpenFOAM simulations predict far less cavitation vapor. Fig. 4b also predicts an earlier cavitation collapse, but Fig. 5b suggests that for case ‘C’, cavitation is incipient but not fully established, whilst the experiment shows fully established cavitation throughout the nozzle. This discrepancy is exacerbated by the higher back pressure; inaccuracy in modeling the cavitation inception point leads to increased discrepancy for this marginal case.

The CONVERGE simulations (Fig. 4c and 5c) still under-predict the total amount of vapor and the cloud axial length, but the discrepancy compared to experiments is quite limited. Peak values next to the walls at $x/D = 1-2$ are satisfactorily reproduced. Bubble lifetime is shorter than in the experiments. The amount of cavitation in case C is higher than in case B, and this is consistent with measurements.

Figures 6 and 7 refer to case ‘G’, where we dissolved krypton in the fuel in place of dissolved air (see Table 1). Fig. 6 shows the measurements of total vapor and cavitation vapor, whilst Fig. 7 shows the distribution of non-condensable gas. The total projected gases (both vapor and non-condensable gas) in the experiment (Fig. 6a) are similar to case ‘A’ (Fig. 3a) with the same boundary conditions, but no dissolved gas. This suggests that the presence of the dissolved gas has not
substantially altered the overall amount of vapor in the nozzle. Considering the projected vapor as a fraction of the path length (Fig. 6b) shows that most of the vapor is still located along the nozzle wall.

In Figure 6c, we use the Kr fluorescence data to remove the dissolved gas contribution from the total gases, and obtain an experimental result for the void due to cavitation alone, following Eqn. 17. This reveals that the true cavitation region is an approximately annular region in the upstream part of the nozzle; no true cavitation persists beyond \( x/D = 4.5 \). These data suggest that the dissolved gas acts to displace true cavitation, but that this process occurs further downstream in the nozzle owing to the longer time scales associated with the nucleation of dissolved gas bubbles as compared to cavitation bubbles. The time-scales for the dissolved gas dynamics are not well understood.

The CONVERGE simulation (Fig. 6e) underpredicts substantially the formation of vapor (true cavitation), while most of the void is generated by the local expansion of non-condensable gas (Fig. 7d) which is basically responsible for the whole void fraction predicted by the model. In Fig. 7d we also notice the occurrence of gas expansion as the flow travels downstream along the nozzle centerline, similarly to experimental observation. The gas expansion at the nozzle entrance is instead largely overestimated and this hinders the modeled vapor formation and expansion.

Conclusions

In this paper, we have reviewed two applications of the homogeneous relaxation model for cavitating nozzle flow. The effects of non-condensable gas have been considered, and the data have been compared against novel x-ray fluorescence measurements of the flow in a 500 \( \mu \)m diameter submerged nozzle.
In the absence of non-condensable gas, we find that both models under-predict the total amount of cavitation. Nucleation sites caused by imperfections in the experiment may account for some of this discrepancy. However, the overall structure of the cavitation is well matched, with separation of the cavitation zone from the wall observed.

Considering a limited number of experimental cases, we find that the reduction in cavitation vapor with increasing Reynolds number is captured well, but the vapor distribution at higher cavitation number is not. Again, hysteretic effects in cavitation inception in the experiment will be different than in the model, with the variations becoming more pronounced as cavitation is more incipient.

In the x-ray fluorescence experiments, we have substituted krypton for dissolved air in the fuel surrogate, and have demonstrated for the first time a quantitative measurement of the gas distribution whilst the nozzle is strongly cavitating. In the experiment, we note a marked decrease in the amount of true cavitation vapor relative to the de-gassed case, whilst the total amount of vapor is relatively constant. The non-condensable gas appears to expand to fill the void created by the cavitation, but this process takes some time owing to the order of magnitude difference in nucleation time-scales. Comparing to the simulations, we note substantial variation in the predicted cavitation vapor.

Both models predict an accumulation of non-condensable gas along the nozzle centerline, as observed in the experiments, but with different structure. The prediction of cavitation behavior in the absence of non-condensable gas demonstrates the efficacy of the HRM, however the results reveal that there is some work to do to be able to capture both incipient cavitation and the behavior of non-condensable gas.

Overall, we believe that both codes could be improved if an additional sub-model for the dissolved gas dynamics is introduced, allowing for a proper estimation of the nucleation time-scales.

Figure 5. Experimental and numerical measurements of projected cavitation vapor $m_{cav}$, for Case ‘C’, $P_1 = 12$ bar, $P_2 = 3$ bar, no dissolved gas.
(a) Experimental measurement of total projected gasses $m_{\text{cav}} + m_{\text{Kr}}$.

(b) Experimental measurement of total projected vapor normalized by path length; $(m_{\text{cav}} + m_{\text{Kr}})/\delta z$.

(c) Experimental measurement of projected vapor only due to cavitation, $m_{\text{cav}}$.

(d) Projected cavitation vapor $m_{\text{cav}}$ from OpenFOAM simulation.

(e) Projected cavitation vapor $m_{\text{cav}}$ from CONVERGE simulation. Note the reduced color bar scale.

Figure 6. Experimental and numerical projections of cavitation vapor for Case ‘G’, $P_1 = 10$ bar, $P_2 = 1$ bar, with Krypton dissolved at a mass ratio of $10^{-4}$. 

Page 11 of 15
These early results reveal the importance of gas dynamics and associated time-scales on accurate modeling.

Future work will explore the significance of dissolved gas at higher injection pressures. With higher injection pressures, the transit time through the nozzle diminishes and the available time for dissolved gas to come out of solution is reduced. We will also investigate the potential significance of using krypton, with its higher mass-weighted solubility, as a proxy for dissolved air.

**Acknowledgements**

David P. Schmidt acknowledges the financial support of General Motors Research Center. We gratefully acknowledge the computing resources provided on the Blues and Fusion high-performance clusters operated by the Laboratory Computing Resource Center at Argonne National Laboratory. This research was performed at the 7-BM beam line of the APS at Argonne National Laboratory. Use of the APS is supported by the U.S. Department of Energy (DOE) under Contract
References


